NASA Contractor Report 198507

Computer Simulation of Time-Dependent Effects for Metal Matrix Composites (MMC) Using METCAN

Kurt C. Gramoll
Memphis State University
Memphis, Tennessee

August 1996

Prepared for
Lewis Research Center
Under Contract NGT36–027–011

National Aeronautics and
Space Administration

Computer Simulation of Time-Dependent Effects for Metal Matrix Composites (MMC) Using METCAN

Final Report
1989 NASA-ASEE Summer Faculty Research Program
NASA Lewis Research Center

Kurt C. Gramoll
Assistant Professor
Memphis State University
Mechanical Engineering Department
Memphis, TN 38152

TABLE OF CONTENTS

ABSTRACT

The METCAN (METal matrix Composite ANalyzer) computer code and its underlining theory, including the Multi-Factor Interaction (MFI) equation, were examined for time-dependent response of metal matrix composites (MMC). This study concentrated on modeling time effects for fiber and matrix material properties, particularly for the modulus, and the respective creep response due to thermomechanical loading. The four main concepts addressed were, one, modeling of the three basic stages of creep, two, implementation of the modified MFI equation, three, characterization of in-situ material properties, and four, numerical methods for simulating viscoelastic creep. The difficulty of experimentally obtaining the numerous in-situ material properties for use in METCAN is discussed and two possible alternatives are presented.

INTRODUCTION

The METCAN computer program was developed at the NASA-Lewis Research Center to simulate the response of metal matrix composites (MMC) due to various effects such as temperature, stress, stress rate, temperature rate, fiber/matrix reaction, mechanical cycling, thermal cycling, and time. METCAN uses the MFI equation to model the *in-situ* material properties of both the fiber and matrix, such as their stiffness, coefficient of thermal expansion, and strength. The constants in the MFI equation must be determined experimentally. These models are then used in conjunction with various micromechanic equations developed by Chamis, et al [1], to predict the stress and strain of any general laminate constructed form the modeled fiber and matrix material. The objective of the program is to be able to computationally simulate the response of a product made from MMC by knowing just the basic fiber and matrix *in-situ* material properties.

Up until now the program has not been used for time dependent response even though the basic material model, the multi-factor interaction (MFI) equation, does include a time term. METCAN has been successful in simulating thermomechanical response for single load applications. The purpose of this study is to further investigate time effects in MMC and how to use METCAN to predict those effects. This includes investigating the basic MFI equation, presenting modification to the MFI equation, examining the numerical methods in METCAN and developing new numerical procedures to incorporate time effects, such as creep. It should be noted that plasticity is not included in this study even through it is important to the general response of a MMC at high temperatures. It was felt that time effects should first be studied alone, and fully understood before a viscoplastic theory is introduced.

This study can be divided into four basic areas. The first area is the examination of the basic MFI equation that is used in METCAN. Currently the MFI equation, as proposed by Chamis, et al [1], has only one time term which limits its ability to describe all three stages of creep for a metal matrix composite material. An additional time term is proposed to allow modeling of primary and secondary creep. Furthermore, the new time term incorporates nonlinear temperature and stress effects on the creep rate.

The second area covered in this report, is the actual implementation of the new time term mentioned above into the METCAN code. The modified program is tested on various laminates at several stress and temperature levels. Numerical stability problems that develop for various cases are examined and a new solution method, the Newton-Raphson, is examined as a possible alternative.

The third area deals with the characterization difficulties of *in-situ* material properties. The MFI equation requires the fiber, matrix and the interface between the fiber and matrix to be modeled and characterized separately, and then used in micromechanics equations to generate lamina properties. Difficulties in obtaining *in-situ* material properties, such as experimental limitations and the large number of constants, are discussed. Two possible alternatives, characterization of the orthotropic unidirectional lamina and the use of bulk material properties, are presented.

The last area of research in this study deals with the numerical solution methods currently used in METCAN to calculate the time-dependent response, or creep, of a metal matrix composite. METCAN uses the current values of properties to obtain the response of a metal matrix composite under going thermomechanical loading. Classical viscoelastic theory is presently not available in METCAN. As a direct consequence, METCAN results deviate from the closed form solution of a simple three parameter spring and dashpot model. A new numerical solution method which uses the hereditary concept of creep, i.e. previous stress history effects the current creep strain, is presented as a possible alternative METCAN. This method also uses the free creep strains, similar to free thermal strains, to determine the correct load shifting between the compliant matrix and the stiff fibers due to the mismatch of creep strain.

THE MULTI-FACTOR INTERACTION EQUATION IN METCAN

The MFI equation was developed by Chamis, et al, [1] in an effort to include all possible effects on all material properties. The current form of the MFI equation models the effects of temperature, stress, stress rate, temperature rate, fiber/matrix layer, mechanical cycling, thermal cycling, and time for the matrix, fiber and interface material. The general form of the MFI equation is

$$\frac{P}{P_o} = \left[\frac{T_F - T}{T_F - T_o}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m \left[\frac{\dot{S}_F - \dot{\sigma}}{\dot{S}_F - \dot{\sigma}_o}\right]^\ell \left[\frac{\dot{T}_F - \dot{T}}{\dot{T}_F - \dot{T}_o}\right]^k \left[\frac{R_F - R}{R_F - R_o}\right]^n \cdots$$

$$\cdots \left[\frac{N_{MF} - N_M}{N_{MF} - N_{Mo}}\right]^q \left[\frac{N_{TF} - N_T}{N_{TF} - N_{To}}\right]^r \left[\frac{t_F - t}{t_F - t_o}\right]^s \cdots\cdots \tag{1}$$

where P = A particular material property

T = temperature

S, σ = strength of stress

$\dot{T}$ = temperature rate

$\dot{\sigma}$ = stress rate

R = interface layer reaction or growth

N = cycles

t = time

and subscripts, F = final or maximum property

o = initial property

M = mechanical

T = thermal

The MFI equation is used to model the changing material properties such as stiffness, strength, thermal conductivity, coefficient of thermal expansion, etc. Each material property modeled, does not necessarily use all terms. For instance, the coefficient of thermal expansion is effected little, if any, by the current stress state, however, the stiffness and strength properties are effected by all terms listed above plus possible other effects.

The MFI equation has been used by Chamis, et al, [2,3] to model the temperature and stress effects, and more recently, to model the cyclic loading effects [4]. One of the advantages of the general MFI equation is its modular form which allows adding or deleting terms that are pertinent to the material property under discussion.

In order to better understand the effects of time in the MFI equation, the stiffness property, E, will be used exclusively due to its high dependence on time at elevated temperature. The MFI equation is also simplified to include only temperature, stress and time effects which allows easier presentation of new concepts and ideas. Furthermore, only time, temperature and stress effects have been examined in detail in the literature which limits the experimental verification to only those effects. The simplified MFI equation for stiffness becomes

$$\frac{E}{E_o} = \left[\frac{T_F - T}{T_F - T_o}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m \left[\frac{t_F - t}{t_F - t_o}\right]^s \qquad (2)$$

This basic equation models the stiffness of the fiber, matrix and interface in each of the possible directions, such as radial, transverse, shear or fiber directions. However, each direction (radial, transverse,shear, etc) and material (fiber, matrix, etc) should have different material constants, such as n, t_F, s, σ_o, etc. Even in this simplified form, the enormous number of material constants needed to completely model a composite becomes evident. This section will look primarily at the generic form of Eq. 2 for the stiffness, E, without regard to the direction or material since this general case must be understood and verified before being applied to specific directions and materials.

Basic Form of the Time Term in the MFI Equation

Each of the three terms in Eq. 2 decay as the governing parameter, T, σ or t, gets larger. This type of function will be referred to as a decay function. If each of the terms were plotted verses the their respective governing parameter, i.e. stress, temperature, or time, the form of the graphic would be identical. The general form of the decay function, for an exponent between 0 and 1, reduces slowly at first and then decreases rapidly towards the end, as shown in Fig. 1. The rate in which the function decays is governed by the value of the exponent. Many material properties do exhibit changes similar to decay function, especially for exponent values between 0.5 and 0.1. A good example of this is the effect temperature has on stiffness. At low and moderate temperatures there is only a slight reduction in stiffness but at high temperatures there is a very strong reduction in stiffness, even for small temperature changes, until the melting temperature is reached, where the stiffness essentially becomes zero.

Similar to temperature, most materials have a reduction in stiffness with increasing stress, especially at high stress levels. The decay function for stress in the MFI equation models this effect well if the exponent is small ($0.2 > m > 0.05$). For a stiffening material, a small negitive exponent could be used.

While the stress and temperature terms in the MFI equation are important, the purpose of this study is to examine the time term and thus, the remainder of this report will concentrate on the time effects. Similar to the stress and temperature effects, the time term is also modeled as a decay function. The time effects are evident in any metallic materials at high temperature, and are generally referred to as creep. Creep in metals has three phases; initial, primary and final. The initial phase exhibits a rapid increase in strain but the strain rate is actually decreasing. A strain verses time plot in log-log form would be linear. In the primary or steady state phase the strain rate is constant. This phase is generally the longest and has the largest impact on the total stain of the material. The final phase shows a increasing strain rate and occurs just before failure.

The current form of the decay function for time, $[(t_F - t)/(t_F - t_o)]^s$, can be simplified to $[(t_F - t)/t_F]^s$ if t_o, the initial time, is assumed to be zero. The reduction in stiffness and the associated change in strain is plotted for various exponent values in

Fig. 2 (t_F is assumed to be 100 in all cases). In examination of the strain curves, it becomes evident that this function can model only the final creep phase where the strain rate is increasing rapidly. A major deficiency is the lack of a constant strain rate (or constant slope) at any point of the strain curve.

It could be argued that if the exponent is small, then the curve will be relatively flat and could model the constant strain rate or secondary phase of creep. The drawback in using a small exponent is that the slope or strain rate of the flat portion of the strain curve will be very small or in other words, the curve is horizontal and thus no creep. A small exponent can not accurately describe moderate or high constant strain rates without changing the t_F parameter. A large t_F will allow the relatively flat portion to be nonhorizontal, but now the meaning of t_F has changed; it is no longer the time to failure. Even if t_F is used as a variable to model the stiffness reduction, a constant creep rate can not be maintained for an indefinite period of time. This problem is visually shown in Fig 3 where t_F is allowed to vary when the exponent 's' is held constant.

Another example of the inability of the decay function to model creep is the results of METCAN itself. Figure 4 shows the results of a uniaxial lamina exposed to temperature and axial loading simultaneously. The steady state creep is not present and the final phase is much larger than actual metal matrix composite material response [5,6].

Computer Simulation Using METCAN for Kevlar/Epoxy

To illustrate the difficulties of the MFI equation of not being able to model the primary creep region of a material, METCAN was used to predict the creep strain of a composite constructed from Kevlar 49 fibers and Fiberite 7714A epoxy. These prediction are then compared to actual creep data for a [0/0] laminate. The reason for using a Kevlar/Epoxy system is that experimental data for both the fiber, matrix and laminate are available [7] at various stress and temperature levels to confirm METCAN results.

The normalized creep curve for the Kevlar fibers is shown in Fig. 5 along with two fitted MFI equation curves. In an effort to keep the modeling simple, the stress and temperature effects were not included. Note that the normalized compliance curve was used for curve fitting since only the exponent, s and variable, t_F, need to be fitted. The actual elastic properties of both the Kevlar fibers and matrix can be obtained from handbooks. The best fit for t_F was 22,000 when $s = 0.25$, and $t_F = 40,000$ when $s = 0.5$. Two curves were fitted, each with a different exponent, to identify a possible exponent effect.

The MFI equation does not model the actual creep data for fibers well because the basic form of the MFI equation requires the creep rate to be always increasing which is the exact opposite for Kevlar fibers. The actual data for creep compliance for Kevlar fibers follow a power law, $D(t) = a + bt^{0.04}$, where a and b are material constants and 0.04 is the time exponent. The exponent is the most critical variable in a power law equation and 0.04 for Kevlar fibers has been independently confirmed by Ho, et al. [8].

Similar to the fibers, the creep compliance for the epoxy also follows a power law, $D(t) = a + bt^{0.271}$. The MFI equation is fitted to the epoxy creep curve in Fig. 6 with an exponent of $s = 0.5$ and $t_F = 40{,}000$. Like the fiber equation, the matrix does not fit the decay function because of the basic differences of the MFI and power law equations.

The fitted MFI equation material parameters were used in METCAN to predict a [0/0] laminate creep response with a constant load. The results are shown in Fig. 7. As would be expected, the agreement between actual and METCAN predicted strain is poor since the decay function can not model a decreasing creep rate. It is also interesting to note that the change in exponent and t_F for the fiber had little effect on the total strain.

In conclusion, Kevlar/epoxy composites should not be modeled by the current MFI equation. In order to accurately predict the creep of polymer based composites, the basic material model needs to be able to describe a decreasing creep rate.

Modification of the MFI Equation to Include Primary and Secondary Creep

The most common and simplest method to model creep of a metallic material is to use a linear line to approximate the strain and time relationship such as

$$\varepsilon = \varepsilon_o + at \tag{3}$$

where ε = total stain

ε_o = elastic or initial strain

a = constant creep rate

t = time

This function is a linear line when plotting strain verses time, which models the steady state or secondary creep stage exactly. The associated stiffness reduction is

$$\frac{E}{E_o} = \frac{\varepsilon_o}{\varepsilon_o + at} \tag{4}$$

or if ε_o is assumed to be unity then

$$\frac{E}{E_o} = \frac{1}{1 + at} \tag{5}$$

Both the stain and stiffness are plotted verses time in Fig. 8. The disadvantage of this model is its inability to model the initial and final phases. Equation 5 can also be derived from the decay function. The basic decay function, $[(t_F - t)/t_F]^s$, can be written as $[1 - t/t_F]^s$. by substituting $s = -1$ and $a = -1/t_F$, the decay function becomes Eq. 5. There is no physical meaning for a -1 exponent but the 'a' constant represents the steady state creep rate.

There have been extensions to Eq. 5 to include the initial creep strain (the primary creep region) such as the Andrade's law [9] which can be written in modified form as

$$\frac{E}{E_o} = \frac{1}{1 + \beta t^{1/3} + at} \tag{6}$$

where β is an additional variable that is determined experimentally. The $t^{1/3}$ term has its greatest effect at the initial time period which allows it to model the primary creep phase accurately. The t term becomes dominant at larger times, allowing it to model the secondary creep phase. By using both time terms in the same equation, both the first and secondary creep phases can be model.

The basic decay function, along with the constant creep model, Eq. 5, and the Andrade's model, Eq. 6, are compared to a hypothetical creep curve in Fig. 9. In fitting the decay function model, both the t_F and s parameters were allowed to vary to obtain the best fit. The a and β parameters were fitted for the best constant creep model and for the Andrade's model. The Andrade's model fits the hypothetical creep well for the first and second creep stages but poorly for the third stage. As expected, the linear region matches exactly for the constant creep phase but can not model the first and third creep phases.

On the other hand, the decay model is low at the beginning but does increase rapidly towards the failure time to match the actual creep. However, the decay model will blow up, i.e. increase without bound, if the time is increased only slightly. Obtaining accurate experimental results for the modeling of the final phase is extremely important, but results in the final phases are the least predictable and reliable. In fact most models will not try to model the final phase because of the unreliable and scatter of experimental results. Also note that the decay model uses 2 parameters to fit the model whereas the linear model requires only one, the steady state strain rate. Furthermore, most data in the literature only reports the steady state rate and therefore precludes using models that require knowledge of the initial or final phases.

In order to model all three phases it is recommended that both the decay function and Andrade's model be used together by multiplying the two functions together. The modified MFI equation would become

$$\frac{E}{E_o} = \left[\frac{T_F - T}{T_F - T_o}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m \left[\frac{t_F - t}{t_F - t_o}\right]^s \left[\frac{1}{1 + \beta t^{1/3} + at}\right] \qquad (7)$$

Figure 10 demonstrates the ability of the modified MFI equation to model all three creep phases by varying the variables t_F, s, β, and a. Furthermore, if one of the creep phases is nonexistent for a particular materials, the appropriate variable can simply be set to zero. For instance, if there is no primary creep phase, β is simply set to zero. The flexibility of Eq. 7 does require four variables to be determined experimentally verses the original two in the decay function.

The following section will develop the temperature and stress dependency on creep rate and how it can be modeled. Actual implementation of the the modified MFI equation into METCAN will not be presented until later sections.

Stress and Temperature Dependency on the Creep Rate

It is well known that stress and temperature greatly effect the creep and creep rate of materials, especially metals. Furthermore, the interaction between these effects can be highly nonlinear. The original equation attempts to model this interaction by multiplying the respective decay functions together, Eq. 2. The MFI equation does not allow any of the constants, such as t_F, or exponents to vary with time, stress, or temperature. Certain nonlinear characteristics can be model by multiplication of the terms but not all. To illustrate this point an example will be presented and discussed. For simplicity, only the stress and time parameters will be used in this example. The MFI equation becomes

$$\frac{E}{E_o} = \left[\frac{S_F - \sigma}{S_F - \sigma_0}\right]^m \left[\frac{t_F - t}{t_F}\right]^s \tag{8}$$

where t_o is assumed zero. Also, assume that $\sigma_o = 0$, $S_F = 100$ MSI, $t_F = 100$ min, $E_o = 1$ MSI, $m = 0.5$, and $s = 0.5$. Since this is just a numerical experiment, any acceptable value could be used. Substituting these assumed values into Eq. 8 gives

$$E = \left[\frac{100 - \sigma}{100}\right]^{0.5} \left[\frac{100 - t}{100}\right]^{0.5} \tag{9}$$

This equation requires the stiffness to go to zero in 100 min regardless of stress level or stress history (except when $\sigma = 100$ MSI since the stiffness is already zero). This is not reasonable since a material exposed to a stress of 20 MSI for 20 minutes and then no stress for 80 minutes will not have the same stiffness reduction if it had be exposed to a stress of 90 MSI the full 100 minutes. This implies that the time rate of change of stiffness must be a function of stress. The difficulty with Eq. 9 is that it does not address or model the stiffness reduction rate or creep rate nonlinearity, but only the magnitude of the stiffness. Both the creep rate and stiffness magnitude are function of stress level and must be modeled.

The same numerical experiment could be performed for the temperature effects by looking at only the time and temperature terms of Eq. 2. The results would be similar to the first numerical experiment showing the time term must be a direct function of the temperature level. This implies that t_F, or any other variable that is used to model creep rate such as β or a in the modified MFI equation, will not be a constant but nonlinear function of both stress and temperature.

A computer simulation was also performed using Eq. 1 (METCAN) where t_F was set at 24 hours and the effect of various temperature levels on the stiffness was observed. Figure 11 shows the effect of 1200 F, 600 F and 100 F on the stiffness of a material over time. It is important to note that the failure time, i.e. when stiffness becomes zero, is nearly the same for all temperatures, which is not realistic. The lower temperature simulations should give a much higher stiffness at 24 hours then the other two temperatures. This difficulty can be over come by making one or more of the variables in the time term a function of stress and temperature.

The nonlinear effect of stress and temperature on creep has been investigated for both bulk metallic materials [9,10] and for metal matrix composite materials [5,6]. In both cases, the constant creep rate or the secondary creep phase was modeled successfully by Dorn's equation that uses a Arrhenius relationship for the nonlinear temperature effects and the Norton's eqaution for the nonlinear stress effects

$$\dot{\varepsilon}_s = \exp\left(\frac{-Q}{RT}\right)\left(\frac{\sigma}{\sigma_c}\right)^n \tag{10}$$

where $\dot{\varepsilon}_s$ = steady state creep rate

T = Temperature

σ = Stress

R = universal gas constant, 8.314 joule/mole K°

Q = creep activation energy, a material constant

σ_c, n = material constants

The effect of temperature and stress on the primary and final creep stage is much harder to determine and is generally not modeled. This study will only model the constant creep rate variable 'a' in the modified MFI equation, Eq. 7, as function of temperature and stress, and will use the form of Eq. 10. The other variables, t_F, s and β should also be function of temperature and stress but due to the numerical difficulties and the lack of experimental data, they will remain constants at this time. Substituting Eq. 10 into Eq. 7 gives

$$\frac{E}{E_o} = \left[\frac{T_F - T}{T_F - T_0}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_0}\right]^m \left[\frac{t_F - t}{t_F - t_o}\right]^s \cdot \left[\frac{1}{1 + \beta t^{1/3} + \exp\left[\frac{-Q}{RT}\right]\left[\frac{\sigma}{\sigma_c}\right]^n t}\right] \qquad (11)$$

Equation 10 allows the material to be nonlinear for the creep rate and nonlinear for the stiffness magnitude. Figures 12 and 13 show graphically the effect of temperature and stress for the time terms in Eq. 11.

It is recommended that the modified MFI equation be used to model the time, temperature, and stress effects on the material response of metal matrix composites. Since this equation has two levels of nonlinearity, as compared to only one level for the original MFI equation, the current algorithm used in METCAN might not be stable or converge to the correct answer. Implementation of the modified MFI equation and further investigation in the solution methodology of the METCAN code is done in the following sections.

IMPLEMENTATION OF THE MODIFIED MFI EQUATION

Implementation of the modified MFI equation into the METCAN computer code involved setting up three separate subroutines, one for each of the three materials, fiber, matrix, and interface. Each subroutine used the modified MFI equation, Eq. 7, to model the time, stress, and temperature effects on the stiffness. These three routines, called MECF, MECM, and MECD are used by MECHF, MECHM, and MECHD subroutines, respectively, to calculate the stiffness . The thermal parameters, such as the coefficient of thermal expansion, or the strength parameters were not effected by the program modifications and these material properties continued to use the original MFI equation for modeling stress, temperature, and time effects.

The user must set the equation constants, such as t_F, s, and β, within the subroutine for each material. Also, the user can modify the nonlinear function that models the temperature and stress dependency on the creep rate if required. After the variables and functions are set, the code is compiled and then executed.

METCAN Results Using the Modified MFI Equation

The first set of computer simulation using the modified MFI equation in METCAN were done using SiC/Ti15 material in the METCAN database. The load and temperature were increased linearly form $N_x = 0$ lb/in and $T = 0^\circ$ F to $N_x = 250$ lb/in and $T = 1000^\circ$ F, respectively using

10 steps. This was done in the first one second and then held constant for 90 seconds. In Case 1, both β and a were equal to zero so that only the decay function would be present (i.e. restore the original MFI equation). The decay function variables were t_F = 100, and s = 0.5. This served as a benchmark for the next two cases. In Case 2, β and a were set to constants, 1.0 and 0.01, respectively, to see the effect of a constant Andrade's equation. And in case 3, the complete modified MFI equation, Eq. 11, was used, with β = 1, n = 5, σ_c = 25,000 PSI, Q = 50,000 J/Mole. The strain results of all three cases are plotted in Fig. 14 for a [0/0] laminate with a total thickness of 0.01 inch.

The first two cases predicted constantly increasing but stable creep strains, as expected. However, the third case become unstable after the total load was applied. There were no fiber or matrix failures predicted by METCAN. The instabilities were assumed to be a result of the highly nonlinear Andrade's equation that models stress and temperature effects on the creep rate.

A second set of computer simulations were performed to help isolate the stability problems. These simulations used a creep rate function of the form

$$a = 0.002 \left[\frac{\sigma}{\sigma_c}\right]^n \left[1 + \frac{T}{2000} \right] \qquad (12)$$

for both the fibers and matrix, where σ_c = 50,000 PSI. This form reduced the degree of nonlinearity of temperature, as compared to Eq. 10, which allowed the nonlinear stress effects to be better examined.

Three different cases were simulated, n equal to 1, 3 and 5. The results are shown in Fig. 15. When is $n = 1$ and 3 the solution stays stable but when $n = 5$, the solution becomes unstable. Furthermore, the strains in each of the two plys, which are identical, vary form one to another on the same time step. This indicates the numerical solution process does not check strain compatibility when checking for convergence. The strain should be physically possible as well as converged before the next time step is taken.

An additional difficulty with the solution process is that when METCAN redistributes the load (see SOLUTION METHODOLOGY FOR TIME DEPENDENT MATERIALS) due to creep, the program does not iterate or check for convergence even though the stresses have changed. It is recommended that the METCAN solution process be modified to check strain compatibility and stress equilibrium at each time step before and after the load redistribution.

The numerical stability problems can be minimized by changing all variables in the METCAN code to double precision. When the previous test case where $n = 5$ is run at double precision (or single precision on a Cray computer, which is equivalent to double precision) the instabilities disappear as shown in Fig. 16.

However, if the matrix and fiber materials have different creep rate functions, which is generally the case, the stability difficulties return, regardless of double or single precision. For example, let the creep rate function be

$$a_{fiber} = 0.002 \left[\frac{\sigma}{50,000}\right]^5 \left[1 + \frac{T}{2000}\right] \tag{12a}$$

$$a_{matrix} = 0.02 \left[\frac{\sigma}{30,000}\right]^3 \left[1 + \frac{T}{2000}\right] \tag{12b}$$

for a SiC/Ti15, [0/90] laminate. With a moderate load of only $N_x = 250$ lb/in the prediction become unstable after loading and diverge, as shown in Fig. 17. Instabilities, therefore are not only a function of computer precision but also degree of nonlinearity and laminate layup. The following section will look more closely at the underlying assumption in the solution process used in METCAN and examines other solution methods in an effort to solve the stability difficulties.

One Dimensional Test Case

A simple two layer, one dimensional test case can be use to examine the solution method of METCAN and understand the numerical stability problems. Currently, METCAN uses successive substitution method to determine the stresses in a MMC composite. At each time step, the stiffness is determined from the nonlinear MFI equation and then the stress is calculated. This new stress is then substituted back into the MFI equation to determine a new stiffness and finally a new stress. This cycle continues until the stresses or composite strains converge. This successive substitution method will experience difficulties for moderately to severely nonlinear equation [11,12] and will only converge for a system of equations that are positive definite.

By looking at a simple one dimensional case, the solution method can be isolated from the complexities of the a highly nonlinear two dimensional laminate composite. The one dimensional model consists of two layers, both of which are viscoelastic. The stiffness in both layers are modeled by the following equation

$$E_i = \frac{1}{1 + at} \tag{13}$$

where i is the layer number, t is time and 'a' is a stress nonlinear function (temperature is not included for simplicity). The two layers are in series, as shown in Fig. 18, and are required to have identical strain at all times, similar to lamina layers in laminated composites.

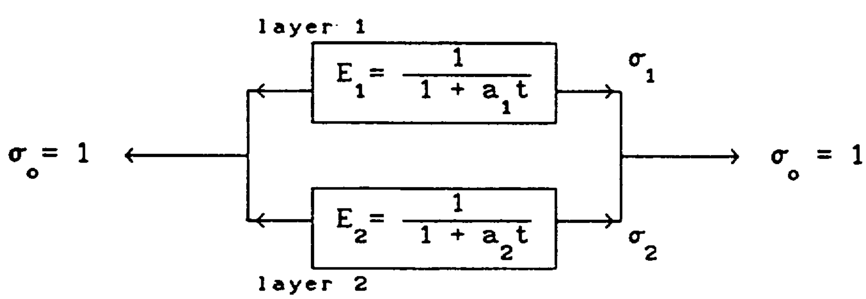


Fig. 18. One Dimensional Two Layer Material.

The 'a' function for the two layers for the particular case to be examined are

$$a_1 = 0.002 \left[\frac{\sigma_1}{0.5}\right]^5 \tag{14a}$$

$$a_2 = 0.02 \left[\frac{\sigma_2}{0.2}\right]^3 \tag{14b}$$

where σ_1 and σ_2 are the stresses in layer their respective layers.

This one dimensional problem was solved using the METCAN algorithm of successive substitution with a time step of 2 and 1. The solution diverges when time is equal to 6 for both time step sizes, as shown in Fig. 19 and 20. The problem was solved using 17 significant places (slightly better than double precision) to greatly minimize any possible numerical induced errors. The same example was then solved using the Newton-Raphson Method [13] to identify if the solution method or the problem formulation was causing the diverging solution. The time step size was 1 for the Newton-Raphson method. The results shown in Fig 21 converge for all solutions steps, and the number of iterations needed for convergence are only two or three, which is substantially less than for the simple substitution method. Furthermore, Newton-Raphson Method will converge for a nonlinear set of equations even if they are not positive definite.

To help eliminate numerical stability problems, and for faster convergence, the Newton-Raphson solution method should be used instead of the successive substitution method currently used in METCAN. Implementation of a Newton-Raphson method will involve reformulating large parts of the current METCAN code, but the final code will be more efficient and more robust.

CHARACTERIZATION OF THE BASIC MATERIAL PROPERTIES

Until now, all discussion has been on the validity of MFI equation from an analytical view point. The MFI equation must also be compared to actual material response tests for the fiber, matrix and interface to verify that this equation can adequately model those material.

One possible method to check the MFI equation is to test a bulk metallic material, such as aluminum or titanium, in uniaxial tests. Uniaxial tests are relatively easy to perform and allow the temperature, time, and stress parameters to be controlled and measured accurately. Through these uniaxial tests each of the material constants can be obtained through a curve fitting process. Statistical methods should also be used to insure the fitting process is within acceptable limits. In this manner, one dimension tests can be accomplished and the results checked in the MFI equation model for stiffness.

The testing method described above assumes two important conditions that may not be true. First, material properties obtained from uniaxial tests are assumed to accurately represent *in-situ* properties of the fiber, matrix and interface. The material *in-situ* can experience geometry, boundary constraints, and chemical interaction effects that might not be detected with uniaxial tests on bulk material. There is evidence in the literature [5,6] that the bulk stiffness property (as a function of stress, temperature, and time) of the fiber and matrix can be used to predict the stiffness properties of a undirectional lamina.

If the *in-situ* material properties were radically different than the bulk properties, then such a prediction of the ply properties would not have been possible.

The second assumption is that the material can be obtained in bulk. For the interface material this is not be possible and thus would prevents uniaxial testing. Furthermore, some material properties might not be obtainable, such as shear and radial stiffness of the fibers due to the limitation of current testing procedures.

These two conditions present a serious problem. The current MFI equation requires the use of only *in-situ* properties, but they are impossible to obtain if uniaxial tests can not be performed on the material in bulk. Furthermore, even if uniaxial tests are assumed to be accurate tests for *in-situ* properties, some tests are still not possible, such radial strength, or tests on the interface material. Theoretically, all material properties for the fiber, matrix and can be found by testing only unidirectional plys and then backing out the properties. This is not experimentally feasible due to the large number of material constants (10 for each material and direction, 3 materials, 4 directions-orthotropic, giving 120 total constants for the modified MFI equation) that must be backed out. It is very difficult to back out 3 or 4 variables and still satisfy acceptable statistical confidence levels. The number of tests required for 120 variables would be on the order of millions, and a large number of these would be creep tests lasting days or even months. For the complete MFI equation with stress and temperature rates effects, and cyclic loading effects, the complexity becomes unimaginable.

Since all material properties can not be obtained at the micro level, i.e. matrix, fiber, interface, it is proposed that the testing be done at the unidirectional lamina or macro level. This will greatly reduce the number of experimental tests needed to characterize the metal matrix material. Furthermore, because a lamina is a bulk material all tests can be performed. This will also eliminate one level of uncertainty in the design process.

The ultimate goal of characterizing a material at the micro or macro level is to give the design engineer the information needed to design and construct a usable product. If a material is characterized at the micro level, then the engineer must first predict the lamina properties, then predict laminate properties, and finally the product. This entails three levels of prediction which is currently not accepted as prudent engineering practice. One level of prediction is common and necessary in every engineering discipline. Buildings are design with a macro level understanding of steel and concrete. The design engineer of a building does not use the strength allowable of rocks, cement, and sand to predict the strength of concrete and then use these calculated concrete values to design a building.

An understanding of the micro level is very important to the material scientist or engineer of new materials to improve properties of materials. But once a new material is proposed, a comprehensive test program is needed to provide macro level design allowables for use in designing the final product. Even for the well understood elastic properties of polymer based composites only macro material properties are used in the design of products such as rocket motor cases, tennis rackets or control surfaces of aircraft.

It is recommended that the computer simulation program for metal matrix composites, METCAN, use lamina material properties for designing laminates and final products (in conjunction with a finite element code).

SOLUTION METHODOLOGY FOR TIME DEPENDENT MATERIALS

Unlike elastic material response, time-dependent response (also known as creep or viscoelasticity) depends on the overall stress and strain history. Because of this, numerical solution methods must store all previous stress and creep strains for use in calculating future stress and strain.

One of the simplest examples to illustrate this hereditary nature of creep is a simple linear spring and dashpot in series, commonly referred to as a Maxwell element. A typical Maxwell element is shown in Fig. 22. If the spring stiffness is E and the dashpot strain rate coefficient is μ then

$$\varepsilon_s = \frac{\sigma}{E} \qquad \text{and} \qquad \dot{\varepsilon}_d = \frac{\sigma}{\mu} \tag{15}$$

where σ is the total applied load (same in both the spring and dashpot), ε_s is the strain of the spring and $\dot{\varepsilon}_d$ is the strain rate in the dashpot. Since the total strain of the system, ε, is the sum of ε_s and ε_d it can be shown that

$$\sigma + \frac{\mu}{E}\dot{\sigma} = \mu\dot{\varepsilon} \tag{16}$$

which describes the time-dependent response of the Maxwell element. Solving Eq. 16 for a constant load, σ_o, gives

$$\varepsilon = \sigma_o \left[\frac{1}{E} + \frac{t}{\mu} \right] \tag{17}$$

with boundary condition, $\varepsilon = \frac{\sigma_o}{E}$ at $t = 0$

Thus for a Maxwell element under constant load, the creep strain increases linearly. This response is as shown in Fig. 22, where $E = \mu = 1$, for two different constant loads, σ_o and σ_1 $(=2\sigma_o)$.

Now suppose that the Maxwell element is loaded at the lower stress, σ_o, until t_1 and then increased to σ_1. If the hereditary nature of the element is ignored, then only the current stiffness of the element, $E = 1/(1+t)$, would be needed to calculate the current total strain. The total strain after t_1 would then be $\varepsilon = \sigma_1(1+t)$ which is equivalent to the response of the element under the higher load for the entire time. This scenario is shown in Fig. 22 as the non-hereditary strain curve. This is not physically possible since a viscoelastic material that experiences a lower stress as well as a higher stress must have a lower total strain then if the higher load was held constant for the same total time (assuming the higher load in both cases were the same). Furthermore, the non-hereditary strain path in this example requires the instantaneous stiffness at t_1 to be less than at t_o. Instantaneous stiffness should not change as a result of creep.

A more realistic approach is to account for the creep strain caused by each of the stress levels and then add the individual effects together as required by linear and nonlinear viscoelastic theory [14]. Using this concept, the total strain for $t \geq t_1$ in the above example would be the sum of the creep strain due to σ_o for the total time and the creep strain due to $\Delta\sigma$ $(=\sigma_1-\sigma_o)$ for the time after t_1. This can be written in equation form as

$$\varepsilon = \sigma_o\left[\frac{1}{E} + \frac{t}{\mu}\right] + \Delta\sigma\left[\frac{1}{E} + \frac{t-t_1}{\mu}\right] \tag{18}$$

In terms of the compliance function S, Eq. 18 is simply

$$\varepsilon = \sigma_o\ S(t) + \Delta\sigma\ S(t-t_1) \tag{19}$$

This is also plotted in Fig. 22 and gives a realistic total strain for the Maxwell element after the second, higher load, σ_2 is applied. This method is commonly called 'strain-hardening'. Also the elastic strains at both t_o and t_1 are the same, and thus the instantaneous stiffness did not change as theory requires. Equation 19 can be generalized for an infinite number of stress steps as

$$\varepsilon = \sigma_o\ S(t) + \sum_{i=1}^{n} \Delta\sigma_i S(t-t_i) \tag{20}$$

where n is equal to the total number of load steps and $\Delta\sigma_i = \sigma_i - \sigma_{i-1}$. For a continuous changing load condition, Eq. 20 can be written in integral form as

$$\varepsilon = \sigma_o\ S(t) + \int_o^t S(t-\tau)\ d\sigma \tag{21}$$

or in the more common form

$$\varepsilon = \sigma_o S(t) + \int_o^t S(t-\tau) \frac{d\sigma}{d\tau} d\tau \tag{22}$$

Equation 22 is referred to as the hereditary integral and is a type of integral equation.

METCAN and the Hereditary Effect of Stress History

Currently METCAN does not separate the creep strain and total strain, nor does it use the stress and stain history or the hereditary integral to calculate the current stress and strain state. Instead METCAN uses the Multi-Factor Interaction (MFI) to model the change in stiffness, E, due to various effects [2,4] at each point in time, independent of previous history. This MFI equation for only temperature, stress and time is

$$\frac{E}{E_o} = \left[\frac{T_F - T}{T_F - T_o}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_o}\right]^m \left[\frac{t_F - t}{t_F - t_o}\right]^s \tag{23}$$

The solution process in METCAN will iterate at each time step until the laminate strains converge using the assumption that a changing instantaneous stiffness correctly models creep strains. The instantaneous stiffness should not vary with time, and the creep strains should be kept separate from the elastic strains.

Figures 23, 24 and 25 show the predicted strain results using METCAN for [0/0], [45/-45] and [90/0] laminates made from SiC/Ti15 subjected to constant stress. Each laminate had three different loading patterns; Case 1: $N_x = 1$, Case 2: $N_x = 2$, and Case 3: $N_x = 1$ until $t = 10$ hours and then increased to $N_x = 2$. The total strain in the load direction (xx-direction) of the laminate is shown in the figures. All three laminates show the total strain for case 3 type loading equal to the case 2 type loading after 10 hours. As discussed previously, this is a result of not using the stress history when calculating the current total strain. One of the main concepts of viscoelasticity is that the instantaneous stiffness will not vary directly with time (possibly indirectly through damage or aging) which is violated with METCAN and thus cases 2 and 3 are equal after 10 hours. Similar problems will develop if the temperature was varied instead of the stress parameter.

The inadequacies of the non-hereditary solution method in METCAN is also evident when a constant load is applied to a composite for a length of time and then completely removed. The METCAN predicts that the strain will return to zero after the load is removed (Fig. 26) which is contrary to the known response of MMC [15]. Generally, the strain for MMC, which has under gone creep, will decrease an amount equal to the initial instantaneous response when unloaded and then slightly recover a small portion of the creep strain over an extended period of time. However, the total strain will not be totally recovered and there will be a permanent deformation. The differences between observed creep strain behavior and METCAN predicted results can be partially accounted for by METCAN's neglecting the hereditary effects of past stress states.

To confirm that MMC do behave in the classical viscoelastic manner, an incremental loading test was performed on an unidirectional laminate composite by D. Bartolotta [16] (the specimen was supplied by D. Petrasek and H. Gray of the Advanced Metallics Branch, NASA Lewis Research Center). The test specimen, Tungsten/Kanthal (FeCrAlY) $[0]_4$, was loaded to 45 KSI, held for 100 minutes, increased to 60 KSI, held 100 minutes, increased to 75 KSI, held 100 minutes, and then unload to 0 KSI. The results are shown on Fig. 27. The two main conclusion from the experiment are, first, the instantaneous stiffness is nearly the same for all loading increments, including the unloading, and second, the strain does not return to zero after unloading. Currently neither of these characteristics can be predicted by METCAN. Methods to correctly predict creep strain are presented in the following sections.

Numerical Methods to Calculate Creep for Stable Materials

One method to calculate the correct total strain is to use the hereditary integral directly. Equation 20 gives the integral in summation form which is most useful for numerical methods. The total strain at any point in time is equal to each stress increment multiplied by the compliance for the length of time the load was applied. Written out for a linear material, this becomes

$$\varepsilon(t) = \sigma_o S(t) + (\sigma_1 - \sigma_o) S(t - t_1) + \ldots\ldots + (\sigma_i - \sigma_{i-1}) S(t - t_i) \qquad (24)$$

This represents an exact solution for a discrete load history of

$$\sigma(t) = \sigma_o H(t) + (\sigma_1 - \sigma_o) H(t - t_1) + \ldots\ldots + (\sigma_i - \sigma_{i-1}) H(t - t_i) \quad (25)$$

where H(t) is the heavyside function and t is the final time. In order to evaluate Eq. 24, the stress and the compliance function, S, must be known at all previous time steps. Furthermore, each new time requires the recalculation of the total strain from $t_o = 0$ to the current time. This means, increasing the number of time steps will geometrically increases the solution time. This effectively places a upper limit on the total number of time steps that can be taken.

Equation 24 modified to incorporate nonlinear effects, such as stress. Findley and Khosla [17] have proposed use of a modified superposition principle of the form

$$\begin{aligned}\varepsilon(t) = {} & \sigma_o S(t,\sigma_o) \\ & + \sigma_1 S(t - t_1,\sigma_1) - \sigma_o S(t - t_1,\sigma_o) \\ & + \ldots\ldots \\ & + \sigma_i S(t - t_i,\sigma_i) - \sigma_{i-1} S(t - t_i,\sigma_{i-1})\end{aligned} \quad (25)$$

They successfully used this equation on isotropic materials. Also, Dillard, et al [18], and Gramoll, et al [7], have successfully used this equation on orthotropic materials such as graphite/epoxy and Kevlar/epoxy.

A natural extension to Eq. 25 is the inclusion of nonlinear temperature effects which gives

$$\begin{aligned}\varepsilon(t) = \; & \sigma_o\, S(t,\sigma_o,T_o) \\ & + \sigma_1\, S(t - t_1,\sigma_1,T_1) - \sigma_o\, S(t - t_1,\sigma_o,T_o) \\ & + \ldots. \\ & + \sigma_i\, S(t - t_i,\sigma_i,T_i) - \sigma_{i-1}\, S(t - t_i,\sigma_{i-1},T_{i-1}) \qquad (26)\end{aligned}$$

In order to correctly predict creep strain in METCAN, Eq 26 could be incorporated into the computer code. The nonlinear effects of temperature and stress are accounted for and the hereditary nature of viscoelastic material is not ignored. However, there is one limitation to this method, the material must not have an increasing strain rate. Such a material is considered unstable and does not have a fading memory. The basic decay function in the MFI equation, Eq. 2 or 7, does result in an increasing creep rate. The following section address such materials and proposes a modification to deal with this difficulty.

Numerical Methods to Calculate Creep for Unstable Materials

One of the main conclusions in the previous section was that METCAN should use past stress history in calculating current creep strains as prescribed by classical viscoelastic theory. Numerical methods were outlined for stable materials but they could not be applied to unstable material with increasing creep rates. This section will present a new numerical solution method for unstable materials.

The three parameter spring and dashpot model (maxwell element in parallel with a single spring) that was previously introduced, is a stable material since the creep rate always decreases for a constant load. However, the MFI equation produces an ever increasing creep rate under a constant load, which is considered an unstable material (Fig. 3). If Eq 26 is used to evaluate the hereditary integral for an unstable material the results could oscillate or diverge rapidly from the correct answer.

A good example of the difficulties encountered in trying to use Eq. 26 to evaluate the hereditary integral for unstable materials is a two layer one dimension model, with one material elastic and the other following the decay function, $E = (1-t/t_F)^s$. The solution for $t_F = 100$ and $s = 1$, diverges quickly from the approximate solution, as shown in Fig. 28. The total strain can not physically exceed 1, yet the standard hereditary solution method based on Eq. 24 exceeded this and eventually blows up.

A second example illustrating the limitations of the hereditary integral for unstable materials is once again a two layer, one dimensional material. The first material is elastic, $E_1 = 1$, and the second material follows a time dependent stiffness relationship of $E_2 = 1/(1+0.01t^n)$. This form for E_2 is convenient since both an unstable and stable material can be obtained by simply changing the exponent, n. If $n = 1$ then the basic Maxwell element is recovered and the strain rate is constant for a constant load. If $n < 1$ the strain rate will be constantly decreasing and for $n > 1$ it will be constantly increasing which is an unstable material.

The total strain for this two layer material was solved using Eq. 26 for various n values. If n is larger than one, the total strain exceeds the physical strain limit of 1 and oscillates about the correct answer, as shown in Fig. 29. For n equal to or less than one, the correct answer is obtained. The basic difficulty in using Eq. 26 for unstable materials is that the hereditary integral assumes the material has a fading memory, which is not the case for unstable materials.

If the MFI equation uses the decay function for time, $(1-t/t_F)^s$, then a new numerical solution process must be developed to solve for the total strain. One possible method, to be called the Additive Creep Method (ACM), is to calculate the creep strain, $\Delta\varepsilon_i^c$ for each time segment, Δt_i, using a simple Euler forward approximation. The creep strain for each segment can then be added together to give the total creep strain. This method neglects past stress history since the material is unstable, but if small enough steps are used the results will converge to the correct answer for the decay function. This method still separates the elastic and creep strains, which becomes important for load shifting, as will be explained in the following section. Furthermore, the instantaneous stiffness does not change with time so the elastic strains due to changing loads (especially unloading) can be computed without including creep strains.

The Additive Creep Method (ACM), can be written in equation form for time and stress as

$$\begin{aligned}\varepsilon(t) = \sigma(t_{n-1})\cdot S(\sigma_{n-1},0) &+ \sigma(t_o)\cdot\Big[S(\sigma_o, t_1 - t_o) - S(\sigma_o,0)\Big]\\ &+ \sigma(t_1)\cdot\Big[S(\sigma_1, t_2 - t_1) - S(\sigma_1,0)\Big]\\ &\ \vdots\\ &+ \sigma(t_i)\cdot\Big[S(\sigma_i, t_{i+1} - t_i) - S(\sigma_i,0)\Big]\\ &\ \vdots\\ &+ \sigma(t_{n-1})\cdot\Big[S(\sigma_{n-1}, t_n - t_{n-1}) - S(\sigma_{n-1},0)\Big] \qquad (27)\end{aligned}$$

where S is the compliance function and n is the total number of time steps. Including temperature, T, and rewriting Eq. 27 in summation form gives

$$\varepsilon(t) = \sigma(t_{n-1})\cdot S(\sigma_{n-1}, T_{n-1}, 0)$$

$$+ \sum_{i=0}^{n-1}\sigma(t_i)\cdot\Big[S(\sigma_i, T_i, t_{i+1} - t_i) - S(\sigma_i, T_i, 0)\Big] \qquad (28)$$

The ACM procedure to calculate creep strains was applied to the previous two layer example, with one layer elastic and the other described by the original MFI equation with only the decay function. The numerical results in Fig. 30 show converging solutions for decreasing time steps for two different exponent values. The solution is stable and does not diverge or blow up but caution does need to be taken in choosing a small enough time step. Furthermore, it should be noted that this solution method is not robust and should not be applied to other unstable material models. Figure 31 shows the difficulties in using the ACM procedure for a two layer material with one layer described by $E_2 = 1/(1+0.01t^2)$. The solution is highly dependent on the step size and does not converge to a single answer with decreasing step size.

In summary, the new Additive Creep Method can be used to calculate the creep in a metal matrix composite when using the MFI equation for a changing stress and temperature. This method also eliminates the need to re-evaluate the complete hereditary integral at each new time step.

Internal Load Shifting

In a composite structure where two different materials are bound together there will material property mismatching. This is especially evident for unidirectional composites where the long fibers will generally have a different thermal expansion coefficient then the matrix material. This mismatch will cause stresses to develop without any externally applied load. Similar to thermal expansion, viscoelastic properties are generally different for each component in a composite structure. Furthermore, each direction (transverse, fiber and shear direction), can have different viscoelastic properties then the adjoining layer, especially if the layers are rotated.

This mismatch in viscoelastic properties will cause load shifting from the more compliant components of the composite, such as matrix, which relax quickly, to the stiffer components, such as the fibers. The overall applied load may not change, but internally there can be large changes in stress, affecting both micro level stresses and ply stresses.

The concept of load shifting can best be represented by a simple example of a hypothetical one dimensional, two layer material. One layer is composed of a linear elastic material modeled by a spring with a stiffness of E_1. The second material is a linear viscoelastic material modeled by a Maxwell element, where the spring constant is E_2 and the dashpot constant is μ. This two layer material is shown in Fig. 32 along with the exact solution for the strain and stress response to a constant load. Layer 2, which is viscoelastic, will lose load as it creeps, and the elastic layer 1, will pick up this load loss. Eventually, all the load will be carried by the layer 1.

The results shown in Fig. 32 for the three parameter solid is one of the few closed form solution available in viscoelasticity that model a two layer material. The solution can be derived by first expressing the Maxwell element that models layer 2, as a differential equation which was was done earlier in Eq. 16. Using E_2 and for E in Eq. 16 and rearranging gives

$$\dot{\varepsilon}_2 = \frac{\dot{\sigma}_2}{E_2} + \frac{\sigma_2}{\mu} \tag{29}$$

where ε_2 and σ_2 are the strain and stress, respectively, in layer 2. The strain equilibrium requires $\varepsilon = \varepsilon_1 = \varepsilon_2$ or $\dot{\varepsilon} = \dot{\varepsilon}_1 = \dot{\varepsilon}_2$ and the stress equilibrium requires $\sigma = \sigma_1 + \sigma_2$ or $\dot{\sigma} = \dot{\sigma}_1 + \dot{\sigma}_2$, where ε and σ are the overall strain and stress. Substituting these equilibrium equations into Eq. 29 gives

$$\dot{\varepsilon} = \frac{\dot{\sigma} - \dot{\sigma}_1}{E_2} + \frac{\sigma - \sigma_1}{\mu} \tag{30}$$

For layer 1, the constitutive equation is

$$\sigma_1 = \varepsilon E_1 \quad \text{or} \quad \dot{\sigma}_1 = \dot{\varepsilon} E_1 \tag{31}$$

Substituting Eq. 31 into Eq. 30 and rearranging gives

$$(E_2 + E_1)\mu\dot{\varepsilon} + E_1 E_2 \varepsilon = \mu\dot{\sigma} + E_2\sigma \tag{30}$$

If the overall stress is constant, $\sigma = \sigma_o$, then $\dot{\sigma} = 0$ and Eq. 32 becomes a first order, ordinary differential equation

$$(E_2 + E_1)\mu\dot{\varepsilon} + E_1 E_2 \varepsilon = E_2 \sigma_o \tag{33}$$

Solving Eq. 33 gives

$$\varepsilon = \frac{\sigma_o e^{-t/b}}{E_1 + E_2} + \frac{\sigma_o}{E_2}\left[1 - e^{-t/b}\right] \tag{34}$$

where $b = \dfrac{(E_2 + E_1)\mu}{E_1 E_2}$

with boundary condition $\quad \varepsilon = \dfrac{\sigma_o}{E_1 + E_2} \quad$ at $t = 0$

This closed form solution is exact if the load, σ_o, is constant. This solution can be used to verify the solution techniques used in any numerical method.

The solution technique currently used in METCAN calculates the stiffness at each point in time using the MFI equation and then uses that stiffness to determine the overall strain by applying stress and strain equilibrium. After the total strain is obtained, each individual layer stress is back calculated since their current stiffness is known. It should be noted that all discussions in this report about METCAN assume the "redistribution" option is set to true. If false, there will be no creep strain for a constant load problem since the increment of load will be zero. However, currently there is no convergence test in METCAN after the loads have been redistributed. Further investigation of the convergence criterion of the redistribution option should be done.

There are two ways to compare the METCAN code and its solution technique to the exact solution of the three parameter example developed earlier. The first is to use the actual program but use new fictitious materials that respond like a spring and Maxwell element, and set all Poisson's ratios to zero to decouple the transverse direction from the fiber directions. The three parameter model can be simulated by METCAN if the fiber material is assumed to be elastic and the matrix material to be viscoelastic. By setting all exponents for the fiber material model in the MFI equation to be zero, the fiber will response elastically. Likewise, when all exponents, except for the time, for the matrix material are zero, the matrix will response viscoelastically. To decouple the fiber direction and transverse direction in a standard two dimensional composite, all Poisson's ratios were also set to zero. In this manner, METCAN was able to simulate a one-dimensional, two layer, viscoelastic material.

The second method is to write a simple program using the METCAN solution algorithm and apply it to the current example. The first few steps in the METCAN solution process will be outlined below. The numerical example is based on the two layer, one dimensional problem introduced previously with $E_1 = E_2 = \sigma_o = 1$ and $\mu = 100$. The stiffness function of layers 1 and 2 are $E^1(t) = 1$ and $E^2(t) = 1/(1+.01t)$, respectively, as shown in Fig. 33.

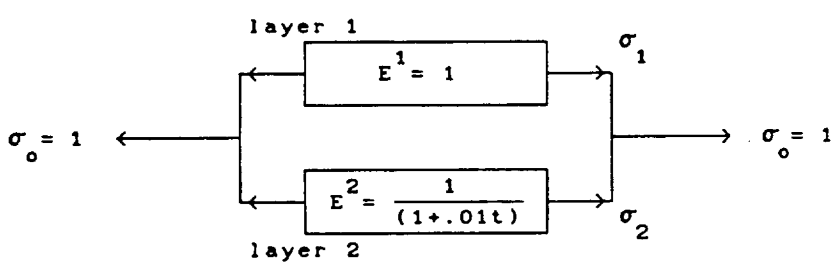


Fig. 33. Simplified Two Layer, One Dimensional Laminate.

For the first step, t = 0, the stiffness of each layer is 1. The total stiffness, E, will then be 2. For a constant load, σ_o, of 1, the total strain will be 0.5 which is correct since no creep strain has taken place at t = 0. The next 5 steps are detailed in Table 1 below.

t	E^1	E^2	$E = E^1 + E^2$	$\varepsilon = \sigma_o / E$	$\sigma^1 = \varepsilon E^1$	$\sigma^2 = \varepsilon E^2$
0	1	1.0000	2.0000	0.5000	0.5000	0.5000
1	1	0.9901	1.9901	0.5024	0.5025	0.4975
2	1	0.9804	1.9804	0.5049	0.5050	0.4950
3	1	0.9709	1.9709	0.5074	0.5074	0.4926
4	1	0.9615	1.9615	0.5098	0.5098	0.4902
5	1	0.9524	1.9524	0.5122	0.5122	0.4878
⋮	⋮				⋮	⋮

Table 1. METCAN Solution of a Spring and Maxwell Element in Parallel.

The total strain calculated from both the actual METCAN code and the METCAN solution method is shown in Fig. 34 along with the exact solution. Both the METCAN solution method and the METCAN program predictions diverge from the closed form solution as time increases. It is important to note that only the numerical method is verified and not the correctness of the material model, i.e. springs and dashpots, or the MFI equation. The spring and Maxwell models are used only because there is a closed form solution available. Furthermore, if a numerical can not converge to the correct answer for a one dimensional case, then it will not converge for a more complex two dimensional case.

An alternate method to numerically simulate load shifting is to treat creep strain as one would thermal strains. Just like thermal strains and stress are not determined by a change in the instantaneous modulus, creep strain should also be calculated separate from elastic strains. Similar to free thermal strain, the free creep strain, ${}^{f}\varepsilon_{i}^{k}$, of each ply, k, is first calculated at each time increment, i. From these free creep strains, an equivalent creep load ${}^{c}N_{i}$ (similar to equivalent thermal load) can be derived. This equivalent load is then applied to the total laminate using the instantaneous elastic stiffness. The resulting global creep strain, ${}^{c}\varepsilon_{i}$ is the actual creep strain for the laminate and each ply, ${}^{c}\varepsilon_{i}^{k}$. For the current problem, ${}^{c}\varepsilon_{i} = {}^{c}\varepsilon_{i}^{k}$, since there is only one dimension, and both layers must have the same strain. This process requires additional work in calculating the free creep strain but the results more closely represent the actual load shifting. This method will be referred to as the Free Creep Strain (FCS) method in this report.

The FCS method was used to solve the same three parameter example previous investigated with METCAN. A flowchart illustrating the numerical solution method is given in Fig. 35. The results for the first few steps are shown in Table 2. The total strain for time steps up to t = 99 is plotted in Fig. 34 along with the exact solution and the METCAN results. The FCS solution method agrees well with the closed form solution for the time step, $\Delta t=1$. This solution also incorporates the hereditary integral to evaluate the free creep strain in each ply as discussed in the previous section.

t	${}^eE_1^1$	${}^eE_1^1$	${}^eE_1$	${}^e\varepsilon_1$	${}^t\varepsilon_1$	σ_1^1	σ_1^2	${}^f\varepsilon_1^1$	${}^f\varepsilon_1^2$	${}^cN_1$	${}^c\varepsilon_1$
0	1	1	2	.5	.5000	.5000	.5000				
1								0.0	.0050	.0050	.0025
	1	1	2	.5	.5025	.5025	.4975				
2								0.0	.0099	.0099	.0050
	1	1	2	.5	.5050	.5050	.4950				
3								0.0	.0149	.0149	.0075
	1	1	2	.5	.5075	.5075	.4925				
4								0.0	.0198	.0198	.0099
	1	1	2	.5	.5099	.5099	.4901				
5								0.0	.0248	.0248	.0124
	1	1	2	.5	.5124	.5124	.4876				
⋮	⋮						⋮	⋮			

Table 2. FCS Solution of a Spring and Maxwell Element in Parallel.

It is recommended that the solution technique in METCAN be modified so that the load shifting between the composite components can be properly calculated. This will help METCAN converge to the correct total strain. One possible method is the FCS method which converges quickly to the correct strain. Furthermore, any method used to account for load shifting should also incorporate the hereditary integral to calculate the creep strain.

CONCLUSIONS AND RECOMMENDATIONS

This study examined the METCAN program code for use in simulating time dependent effects in metal matrix composites, and developed new numerical methods for solving creep strains, and proposed modification to the MFI equation to correctly model all phases of creep. Both the findings and modifications implemented are summarized below, along recommendations for further work.

1) The MFI equation, as currently implemented in METCAN with the decay function for time, can not model the primary and secondary creep stages. A new time term, derived from the Andrade's creep law, was introduced to model all three phases of creep when used in conjunction with the original decay function.

2) The creep rate is not a direct function of stress and temperature in the original MFI equation. The constant creep rate, modeled in the new time term, was made a function of both stress and temperature using the standard Arrhenius equation.

3) The modified MFI equation with stress and temperature dependency was implemented into METCAN. The results indicate a sensitivity to the degree of nonlinearity and the laminate layup. Severely nonlinear models and cross-ply laminates did not converge even at double precision. Convergence problems also developed when the solution method in METCAN, successive substitution, was used on a simple two layer, one dimensional hypothetical composite with moderately nonlinear material properties. When a Newton-Raphson solution method was used, their were no convergence difficulties, and converge was very rapid. It is

recommended that the Newton-Raphson method be implemented into METCAN as the main solution method.

4) The use of *in-situ* material properties for the fiber, matrix and interface was determined impractical because of the current inability of experimental methods to determine such properties. Furthermore, the number of material properties that must be determined are too numerous to be effectively obtained and still stay within accepted statistical bounds. It is recommended that the lamina level be characterized which will reduce the number of material variables and alleviate the need to find *in-situ* properties. An alternative method would be to use bulk material properties for the fiber, matrix and interface, and relate those properties to the *in-situ* properties.

5) The hereditary effects of previous stress on the current strain are not taken in to account in the current solution method in METCAN. Experimental results on unidirectional composites do not agree with METCAN predictions for time dependent response. Basic viscoelastic principles were reviewed and two new numerical procedures were introduced for use on stable and unstable materials. An unstable material is material that has an increasing creep rate with time. These numerical techniques have not been implemented into METCAN but it is recommend to be done in the future.

6) The current method of shifting load from the more compliant matrix to the stiffer fibers due to the time-dependency of the materials in METCAN will cause the total strain to deviate from the true total strain. This was demonstrated by comparing METCAN results with the closed form solution of a three parameter dashpot and spring model. A new solution technique was introduced as a possible method to correctly model the load shifting and applied successful to the three parameter model. This numerical method was not implemented into METCAN due to the time constraints. It is recommended that the method be used in METCAN in the future.

REFERENCES

1. Chamis, C.C. and D.A. Hopkins, "Thermoviscoplastic Nonlinear Constitutive Relationships for Structural Analysis of High Temperature Metal Matrix Composites," NASA TM-87291, 1985.

2. Hopkins, D.A. "Nonlinear Analysis for High-Temperature Multilayered Fiber Composite Structures," NASA TM-83754, 1984.

3. Murthy, P.L.N., D.A. Hopkins and C.C. Chamis, "Metal Matrix Composite Micromechanics: In-Situ Behavior Influence on Composite Properties," 3rd Joint ASCE/ASME Mechanics Conference, Univ of San Diego, CA, July 9-12, 1989.

4. Chamis, C.C., P.L.N. Murthy and D.A. Hopkins, "Computational Simulation of High Temperature Metal Matrix Composites Cyclic Behavior," ASTM Symposium on Thermal and Mechanical Behavior of Ceramic and Metal Matrix Composites, Atlanta, GA, Nov 1988.

5. Petrasek, D.W. and R.H. Titran, "Creep Behavior of Tungsten/Niobiumand Tungsten/Niobium-1 Percent Zirconium Composites," NASA TM-100804, 1988

6. McDanels, D.L., R.A. Signorelli and J.W. Weeton, "Analysis of Stress-Rupture and Creep Properties of Tungsten Fiber Reinforced Copper Composites," Fiber-Strengthened Metallic Composites, ASTM STP 427, 1967, pp 124.

7. Gramoll, K.C., D.A. Dillard and H.F. Brinson, "Thermoviscoelastic Characterization and Predictions of Kevlar/Epoxy Composite Laminates," Composite Materials: Testing and Design, 9th Conf., ASTM 1059, Sam Garbo (Ed.), 1989.

8. Ho, T., R.A. Schapery and B.C. Harbert, "The Viscoelastic Behavior of Kevlar/Epoxy Materials," LTV Aerospace Report, Contract No. DAAG46-83-C-0032, released thru U.S. Army Materials Technology Laboratory, Watertown, Mass., 1985.

9. Rabotnov, Y.N., Creep Problems in Structural Members, North-Holland, 1969.

10. Odqvist, F.K.G., Mathematical Theory of Creep and Creep Rupture, Oxford, 2nd Edition, 1974.

11. Cook, R.D., Concepts and Applications of Finite Element Analysis, John Wiley, 2nd Edition, 1981.

12. Tillerson, J.R., J.A. Stricklin and W.E. Haisler, "Numerical Methods for the Solution of Nonlinear Problems in Structural Analysis," in Numerical Solution of Nonlinear Structural Problems, R.F. Hartung, ed., Vol. 6, AMSE, 1973.

13. Carnahan, B., H.A.Luther and J.O. Wilkes, Applied Numerical Methods, John Wiley, 1969.

14. Flügge, Wilhelm, Viscoelasticity, Second Revised Edition, Springer-Verlag, 1975.

15. Bartolotta, P.A., P.K. Brindley and J.R. Ellis, "Verification of High-Temperature Structural Analysis Procedures," Proceedings of HITEMP Review 1988 Workshop, NASA Lewis Research Center, Cleveland, Ohio, Nov., 1988.

16. Bartolotta, P.A., Unpublished Test Results, NASA Lewis Research Center, Fatigue and Fracture Branch, 1989.

17. Findley, W.N. and G. Khosla, "Application of the Superposition Principle and Theories of Mechanical Equation of State, Strain, and Time Hardening to Creep of Plastics Under Changing Loads," Journal of Applied Physics, Vol. 26, No. 7, 1955.

18. Dillard, D.A. and H.F. Brinson, "A Numerical Procedure for Predicting Creep and Delayed Failures in General Laminated Composites," Long Term Behavior of Composites, ASTM 813, T.K. O'Brain (Ed.), 1983, pp. 23-37.

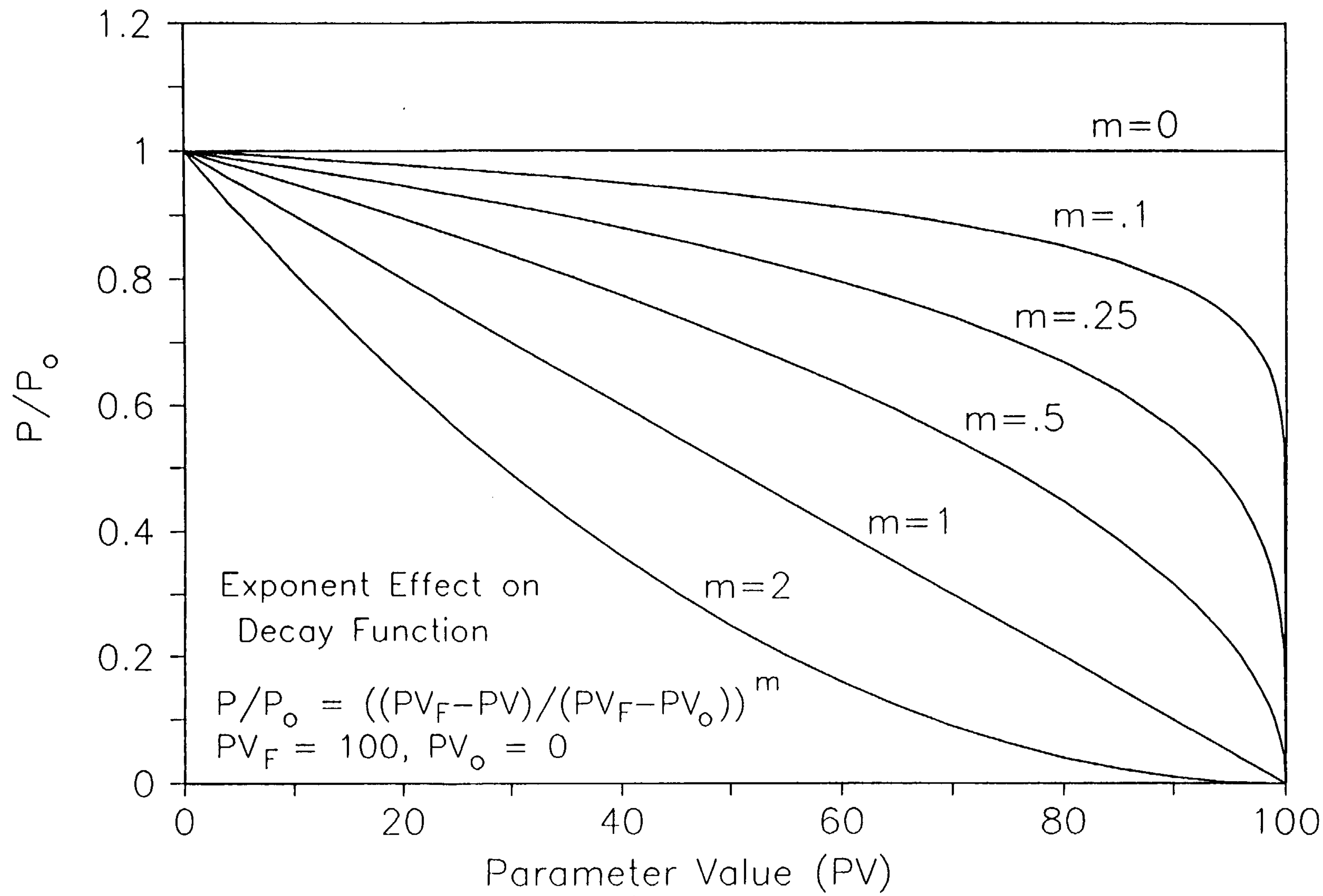


Fig. 1. Decay Function for Various Exponent Values.

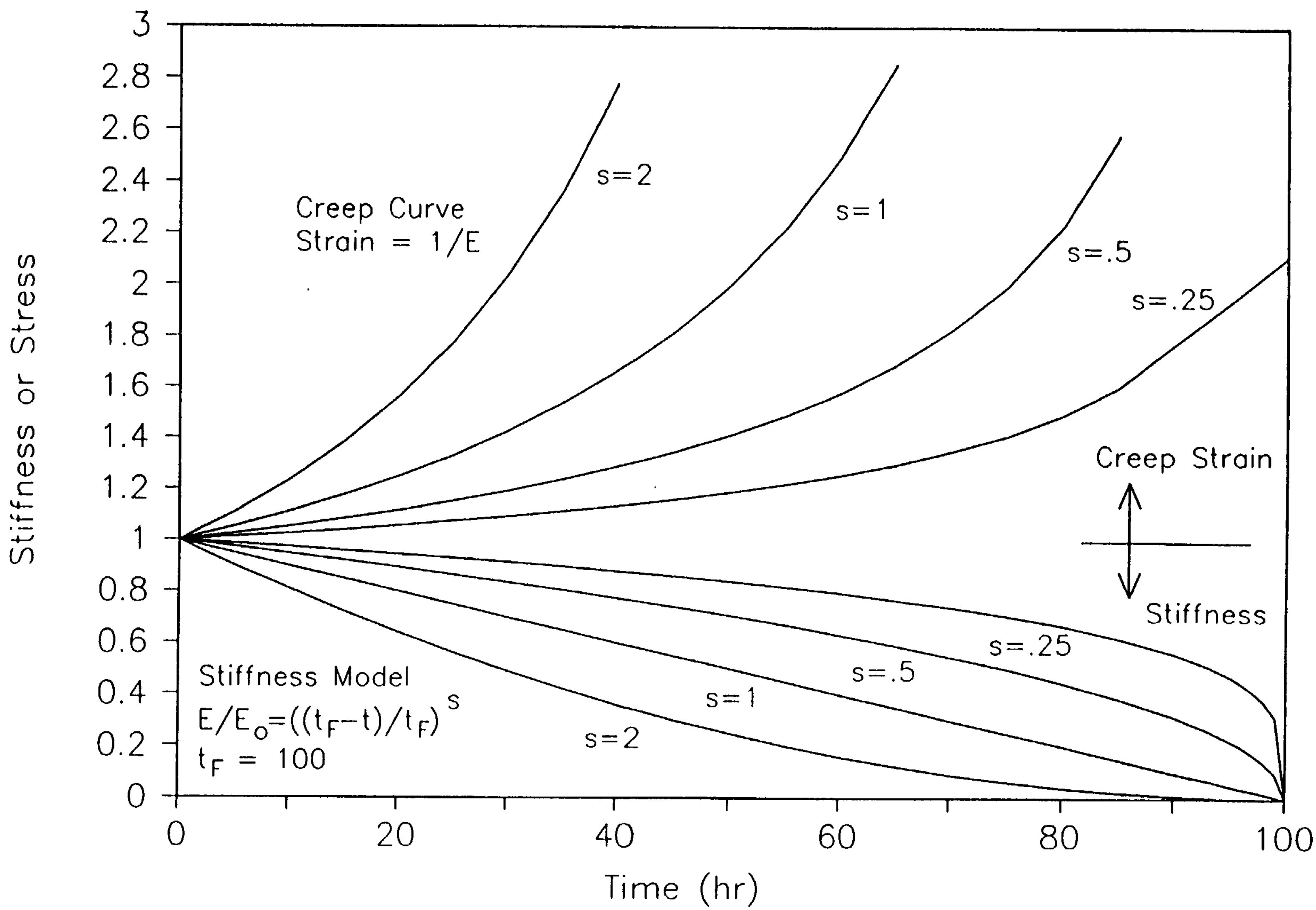


Fig. 2. Stiffness and Creep Curves for Various Values of Exponent s.

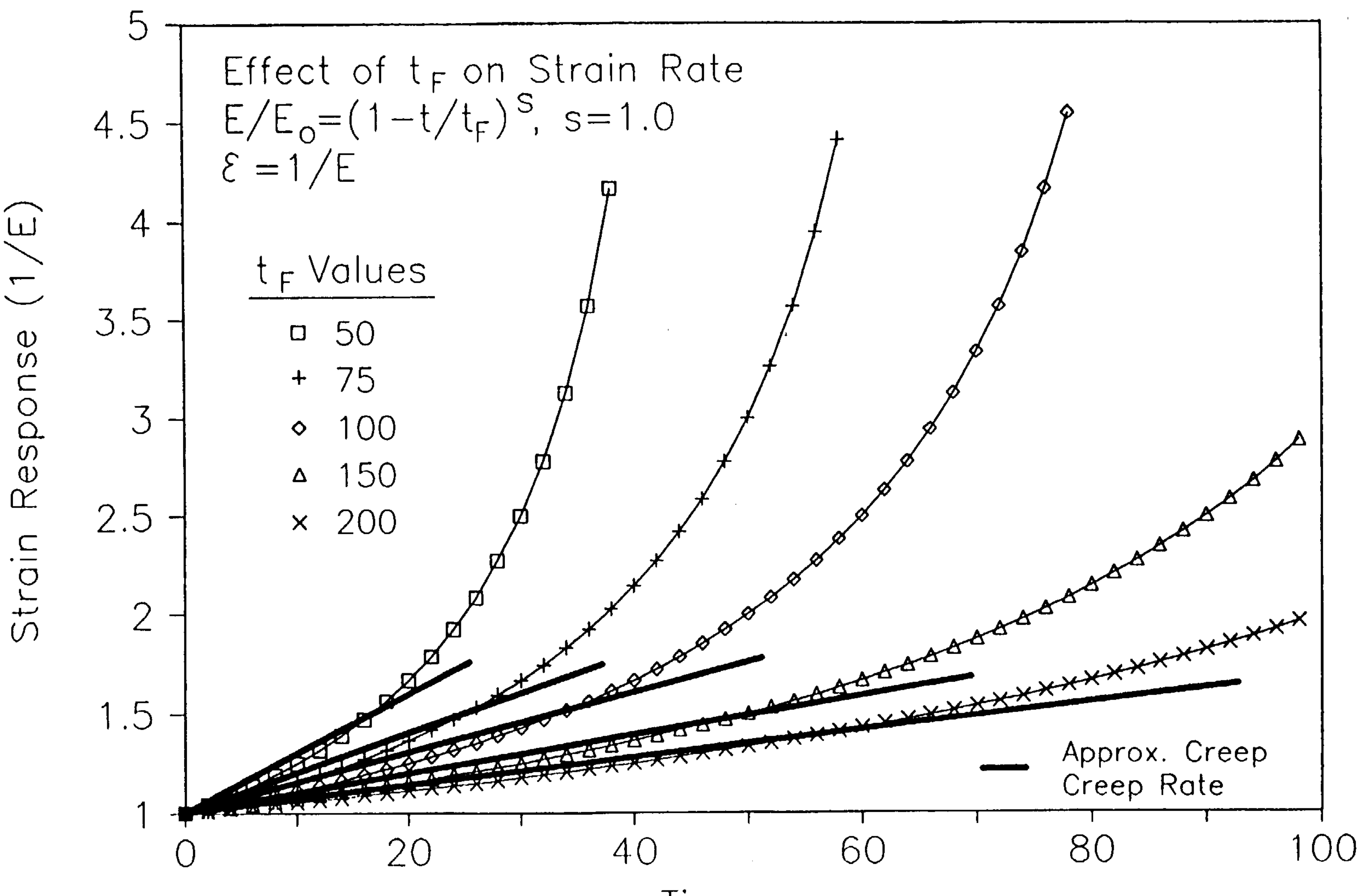


Fig. 3. Effect of Changing t_F with the Exponent Constant.

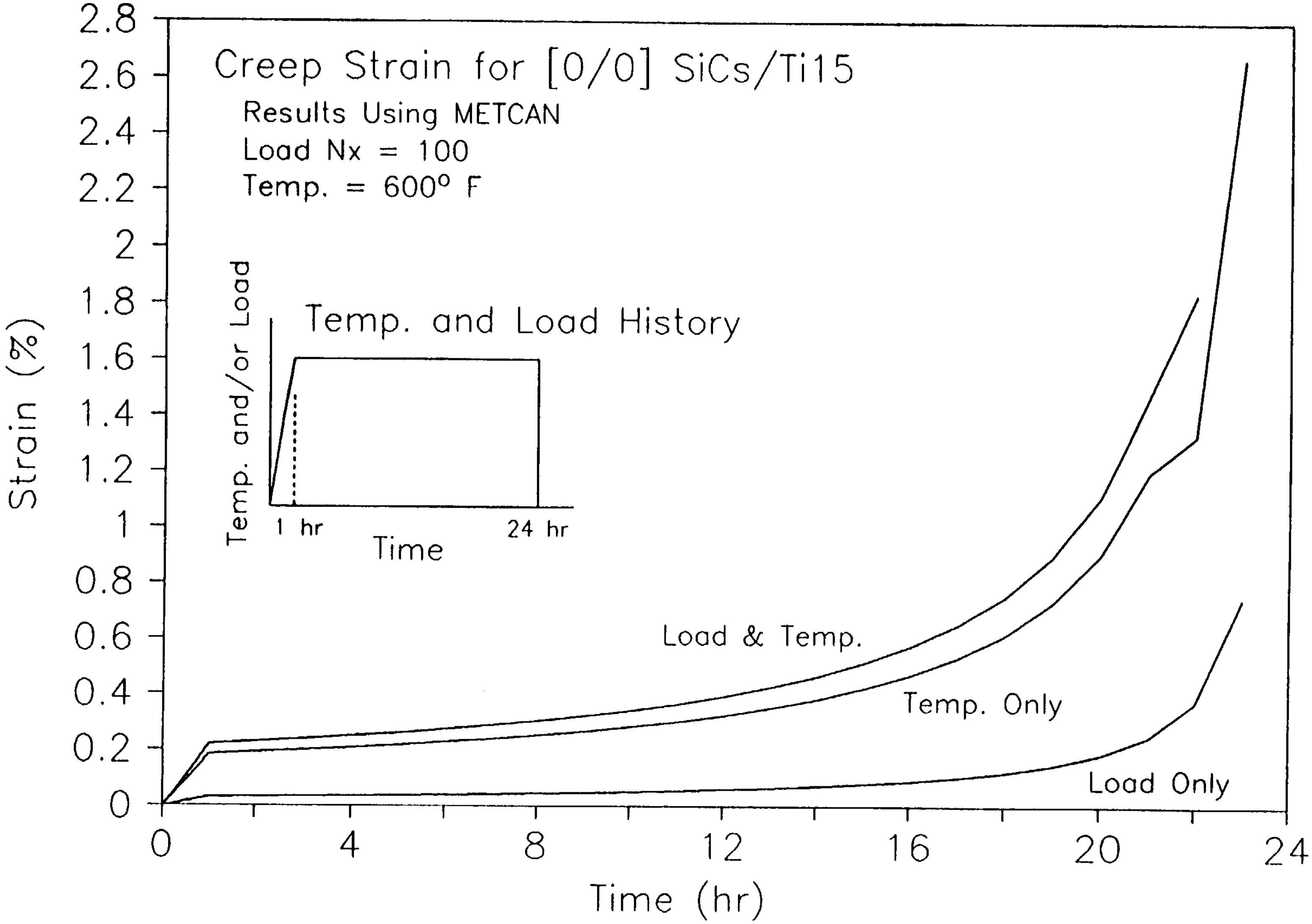


Fig. 4. Actual Test Case Using METCAN with Decay Function.

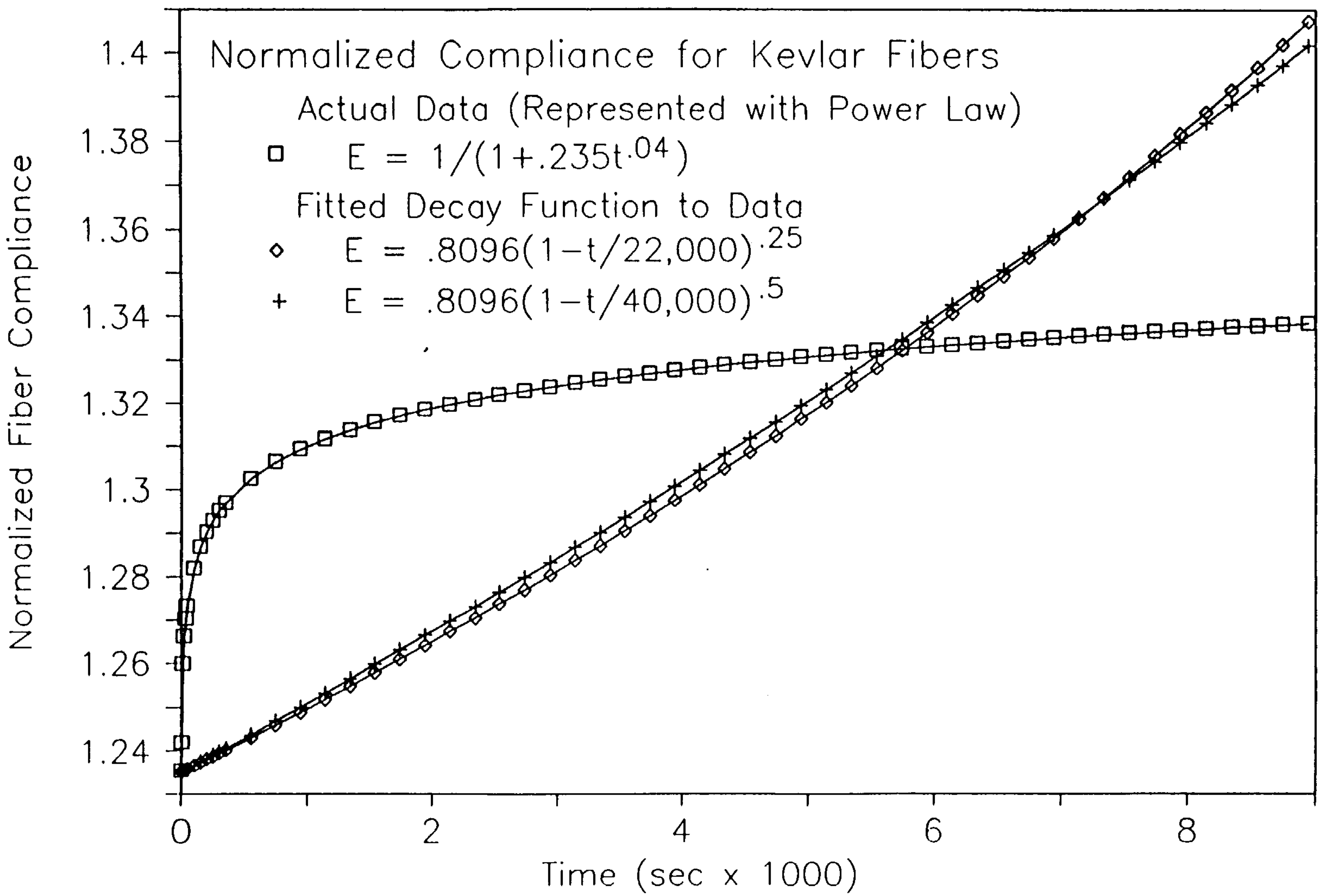


Fig. 5. Compliance for Kevlar Fibers and Fitted Decay Function.

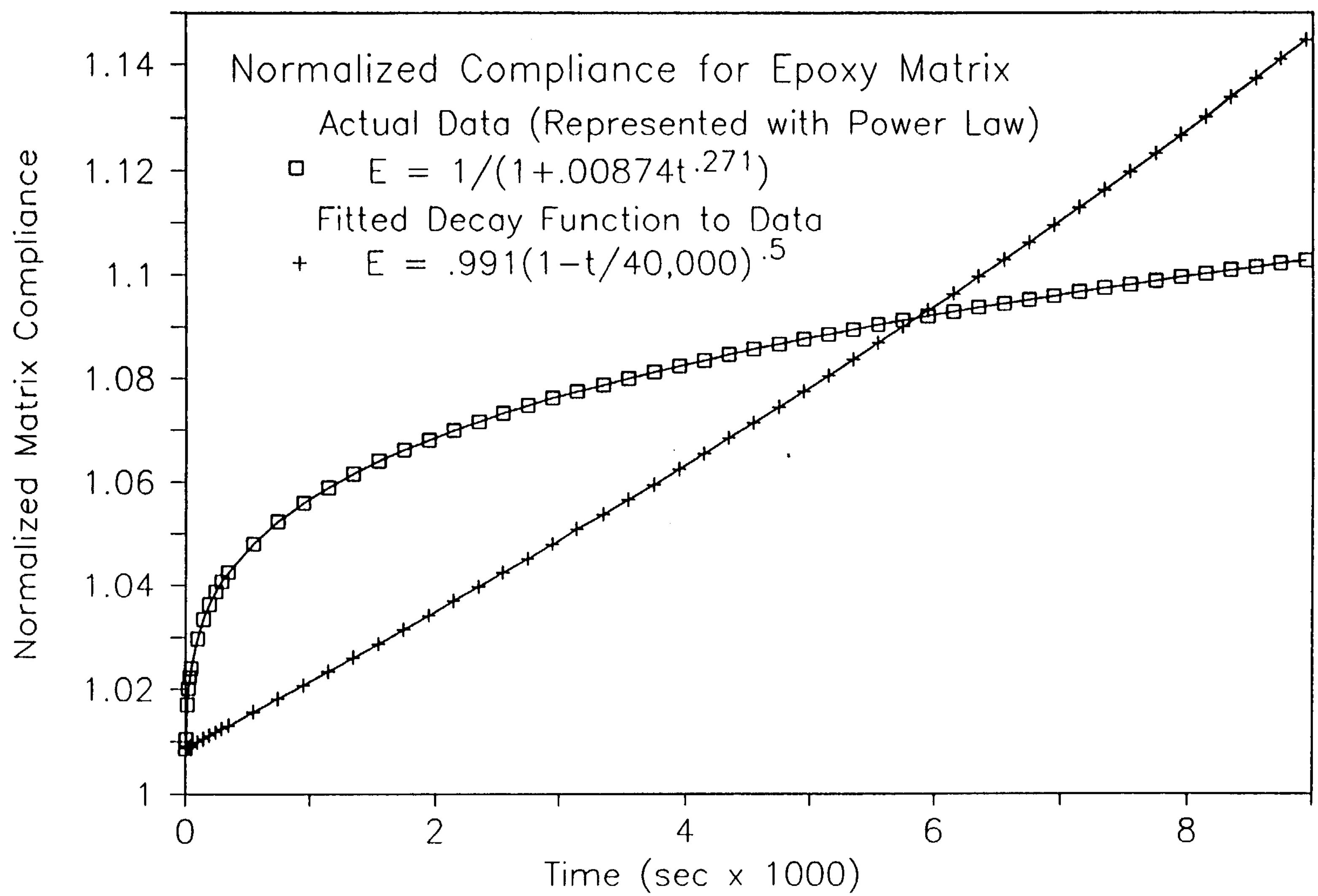


Fig. 6. Compliance for Epoxy Matrix and Fitted Decay Function.

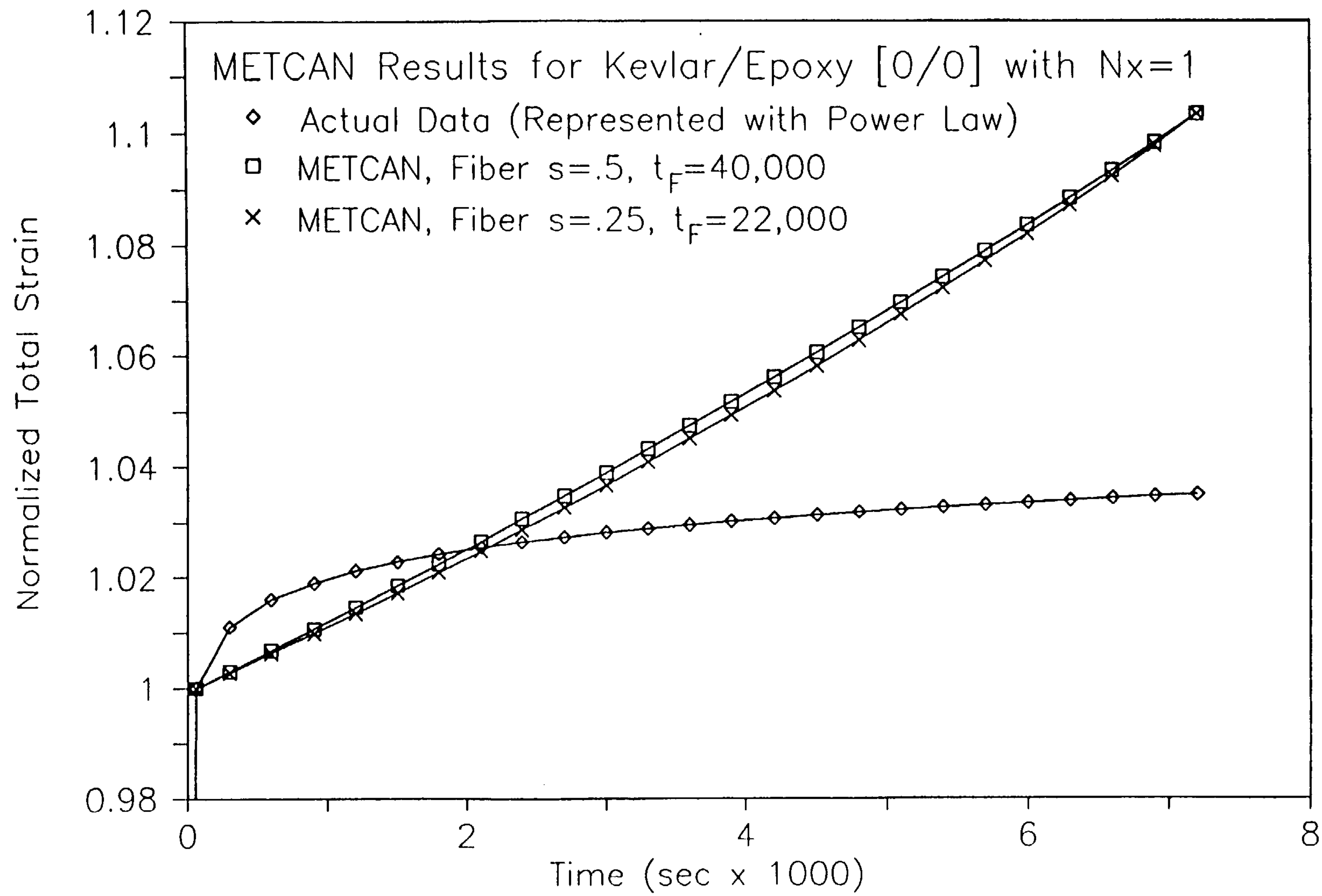


Fig. 7. Compliance for Kevlar/Epoxy Laminate Compared to METCAN Results.

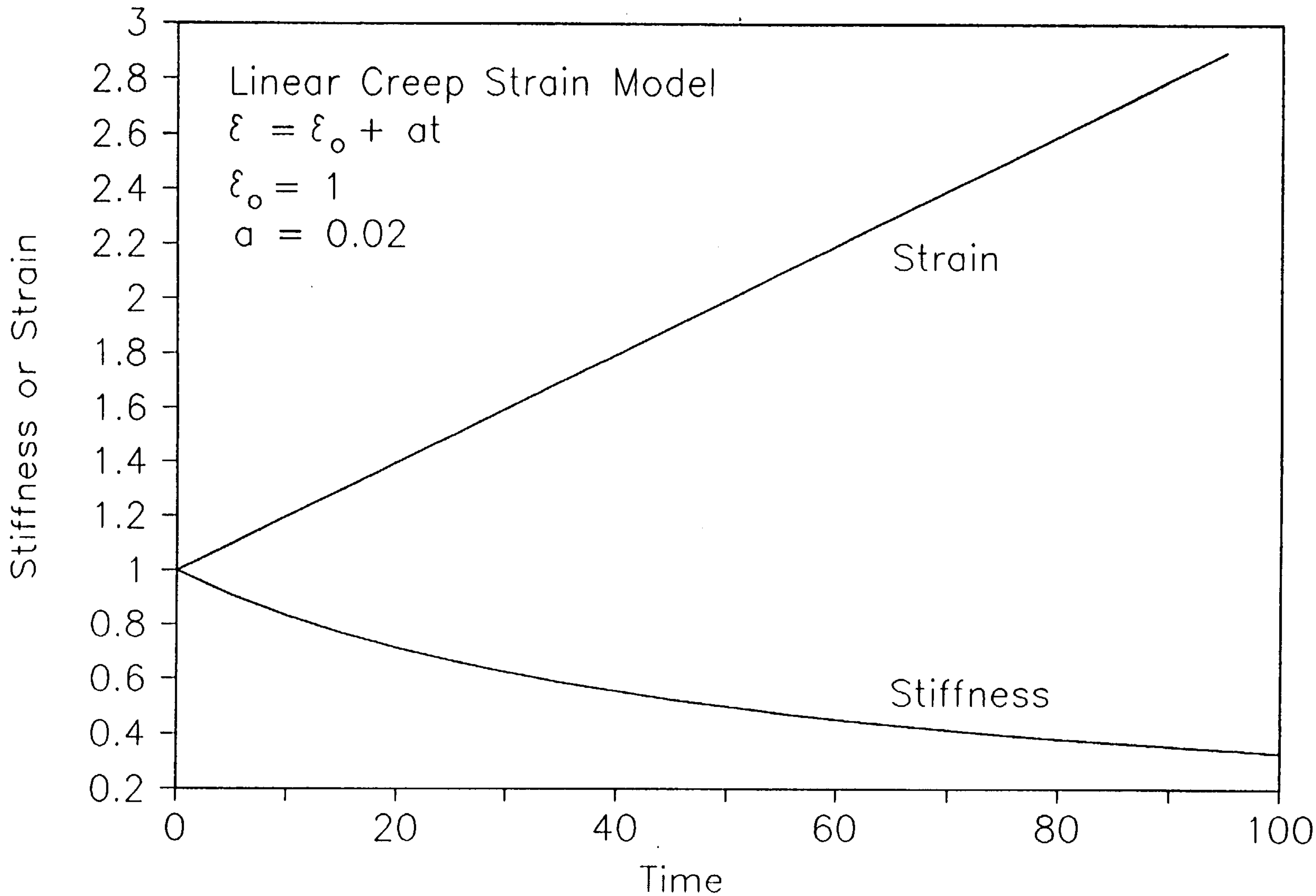


Fig. 8. Creep Strain and Stiffness Using Linear Model.

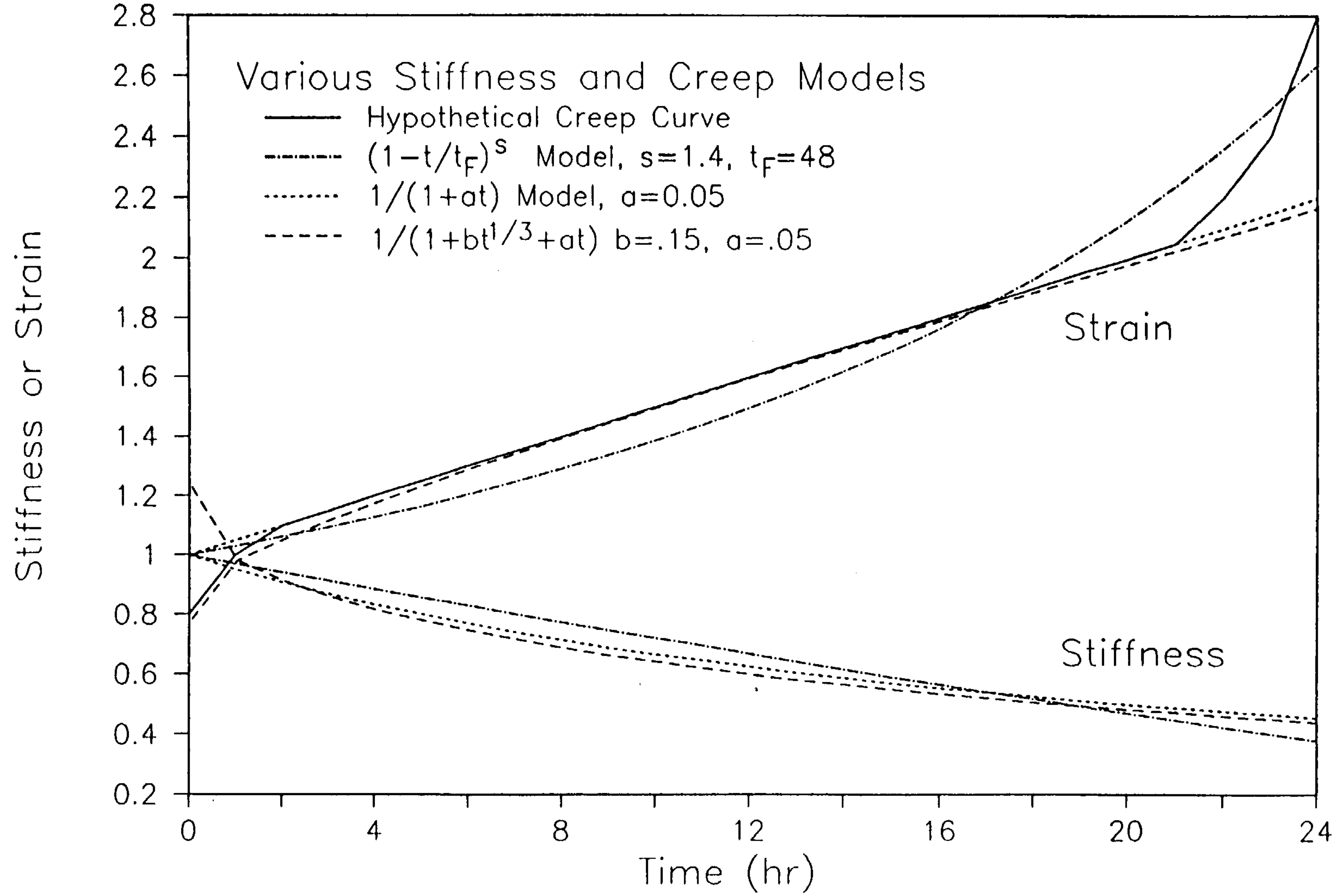


Fig. 9. Comparison of Creep Models.

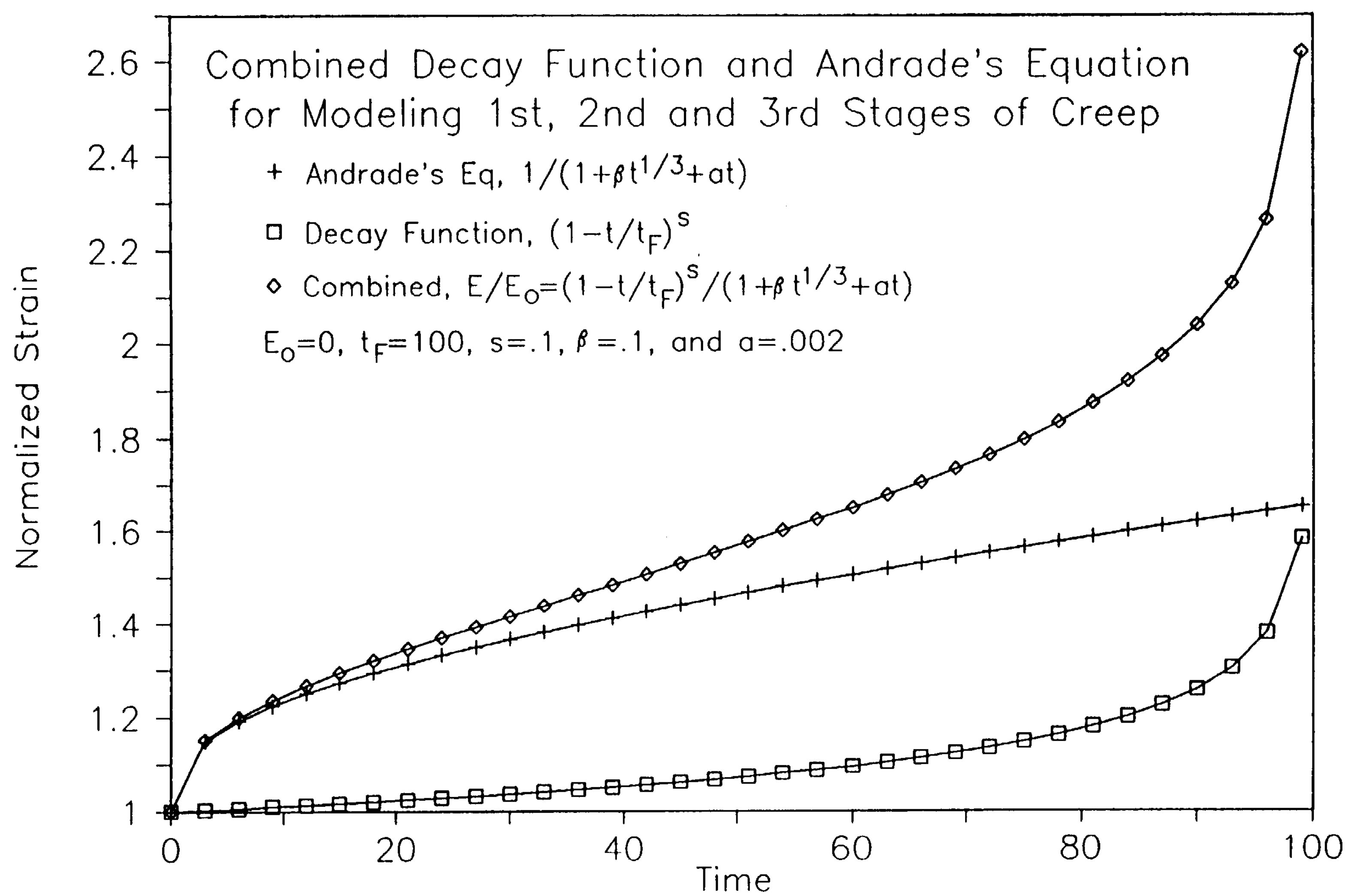


Fig. 10. Strain Using the Combined Deacy and Andrade's Function for Creep.

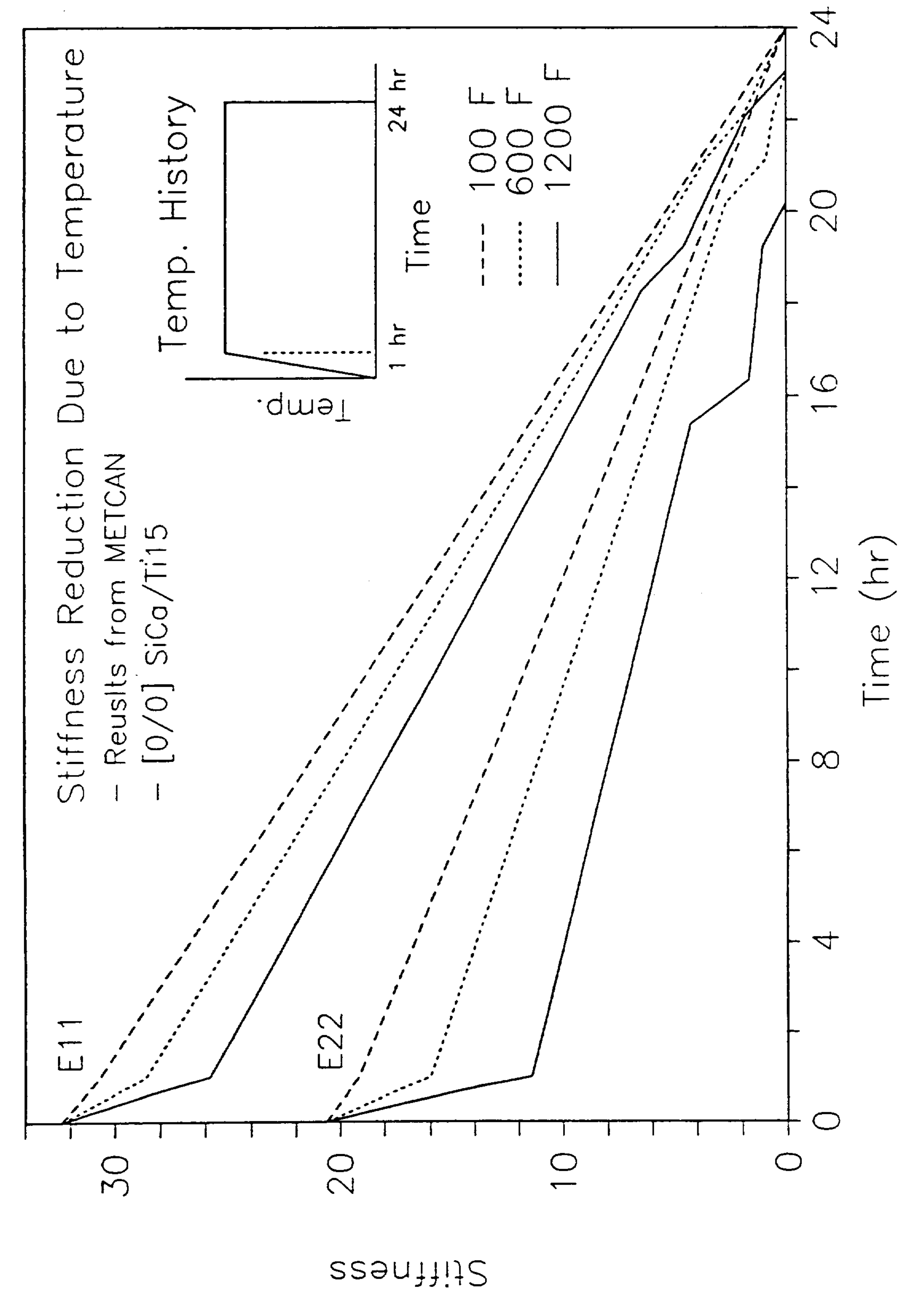


Fig. 11. Change in Stiffness Due to Temperature.

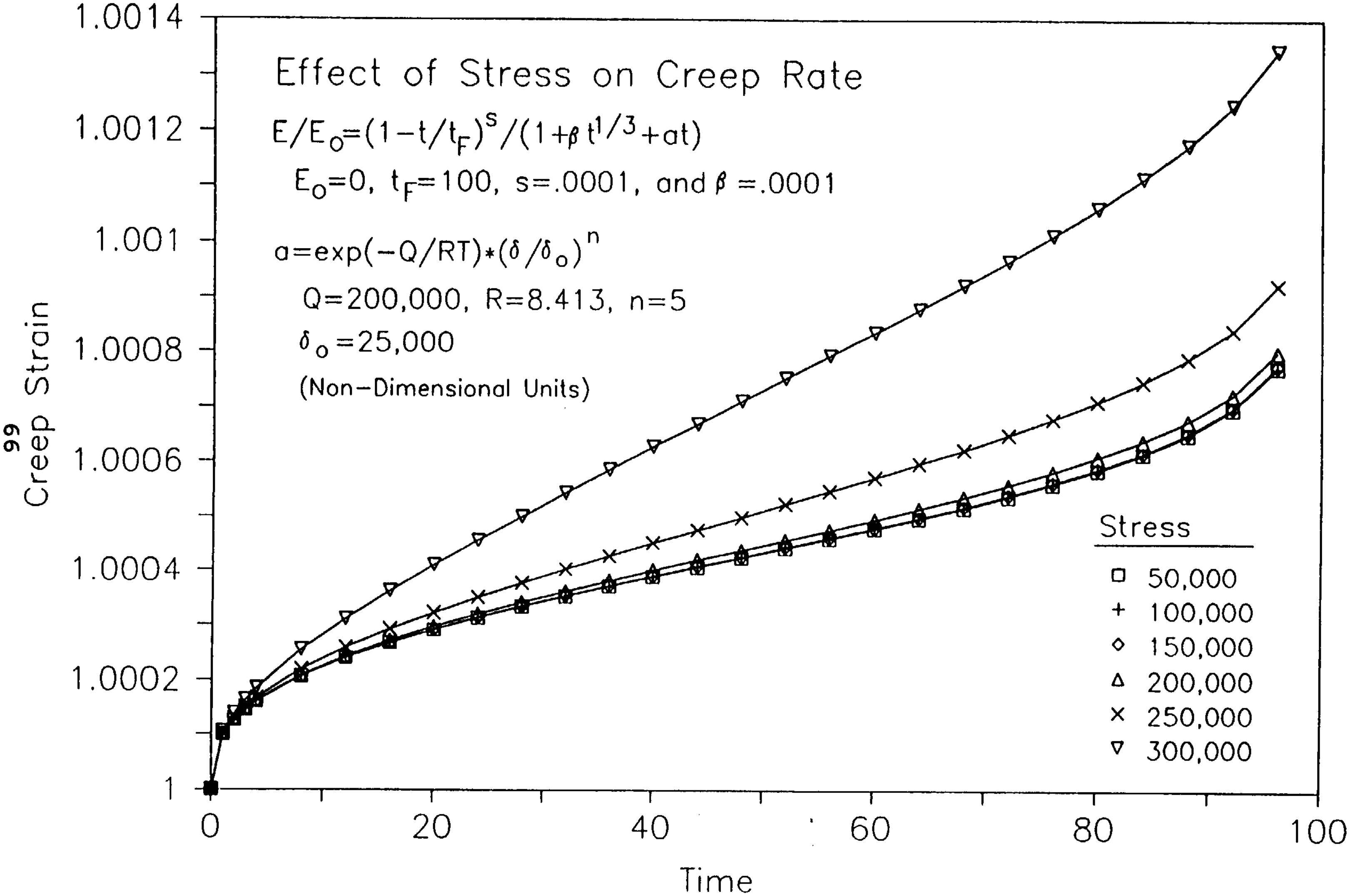


Fig. 12. Effect of Stress on Creep Rate Using the Modified MFI Equation.

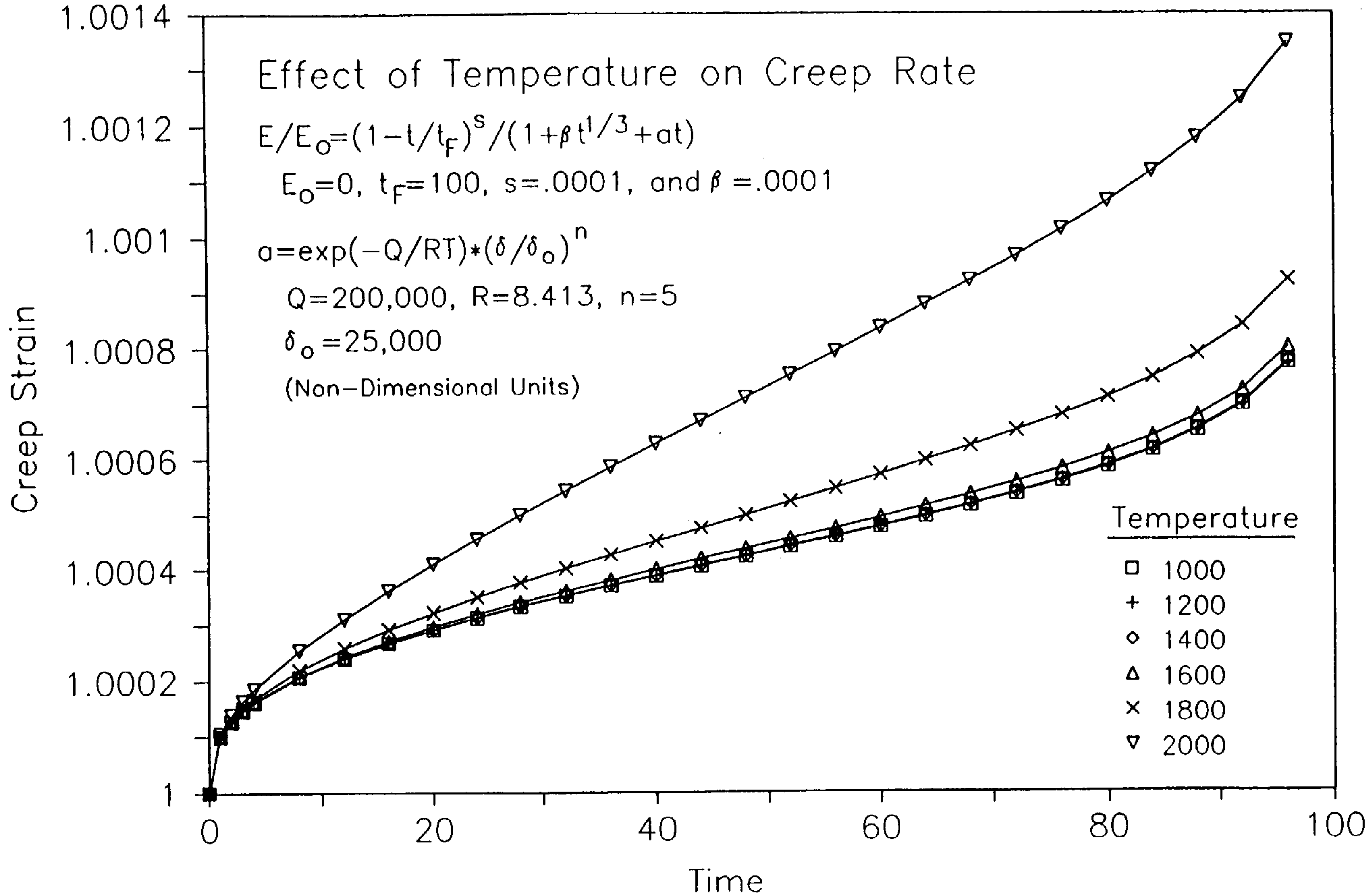


Fig. 13. Effect of Temperature on Creep Rate Using the Modified MFI Equation.

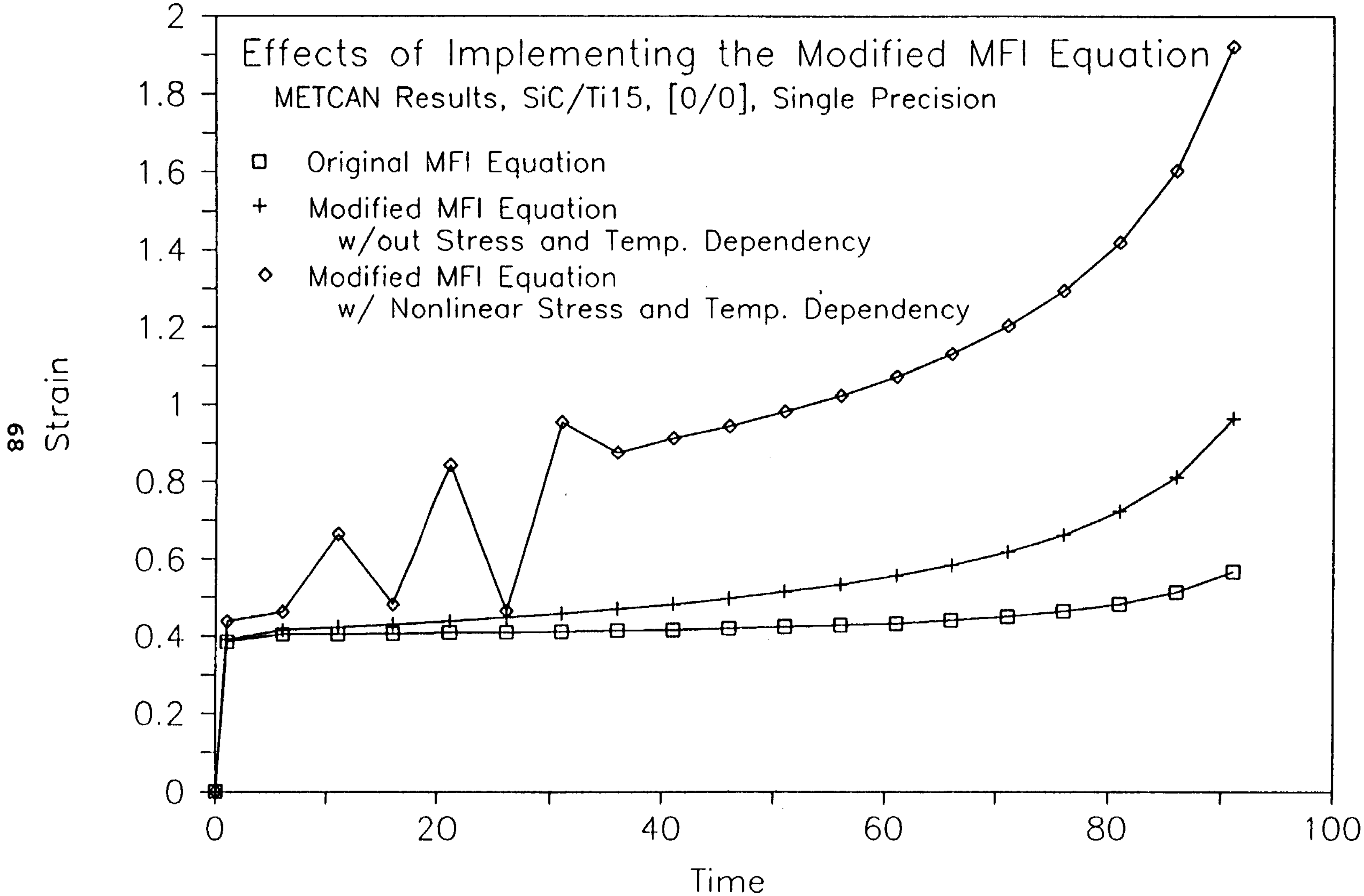


Fig. 14. Stability Difficulties with the Modified MFI Equation.

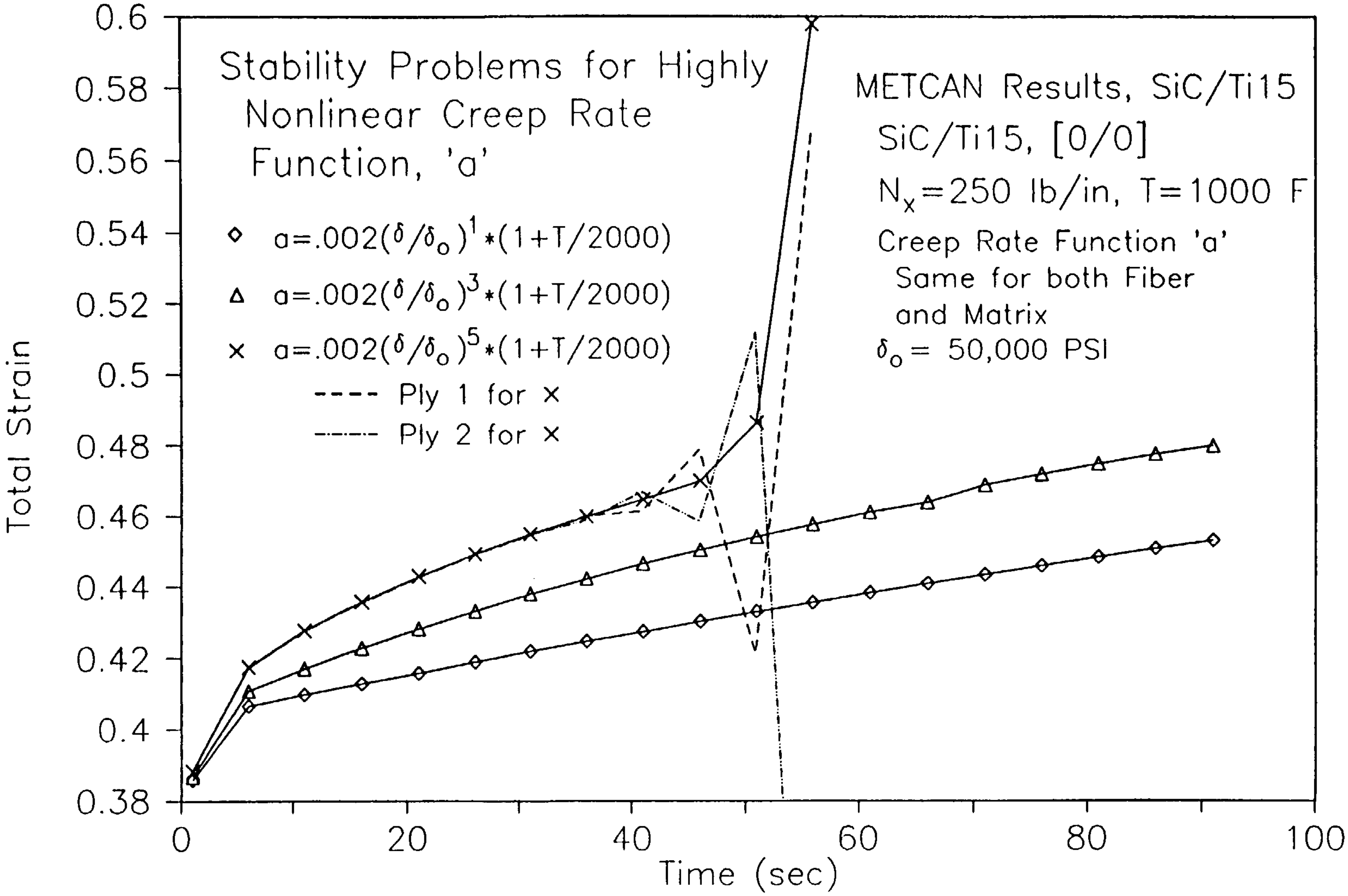


Fig. 15. Various Nonlinear Stress and Temperature Functions for the Constant Creep Rate Term 'a'.

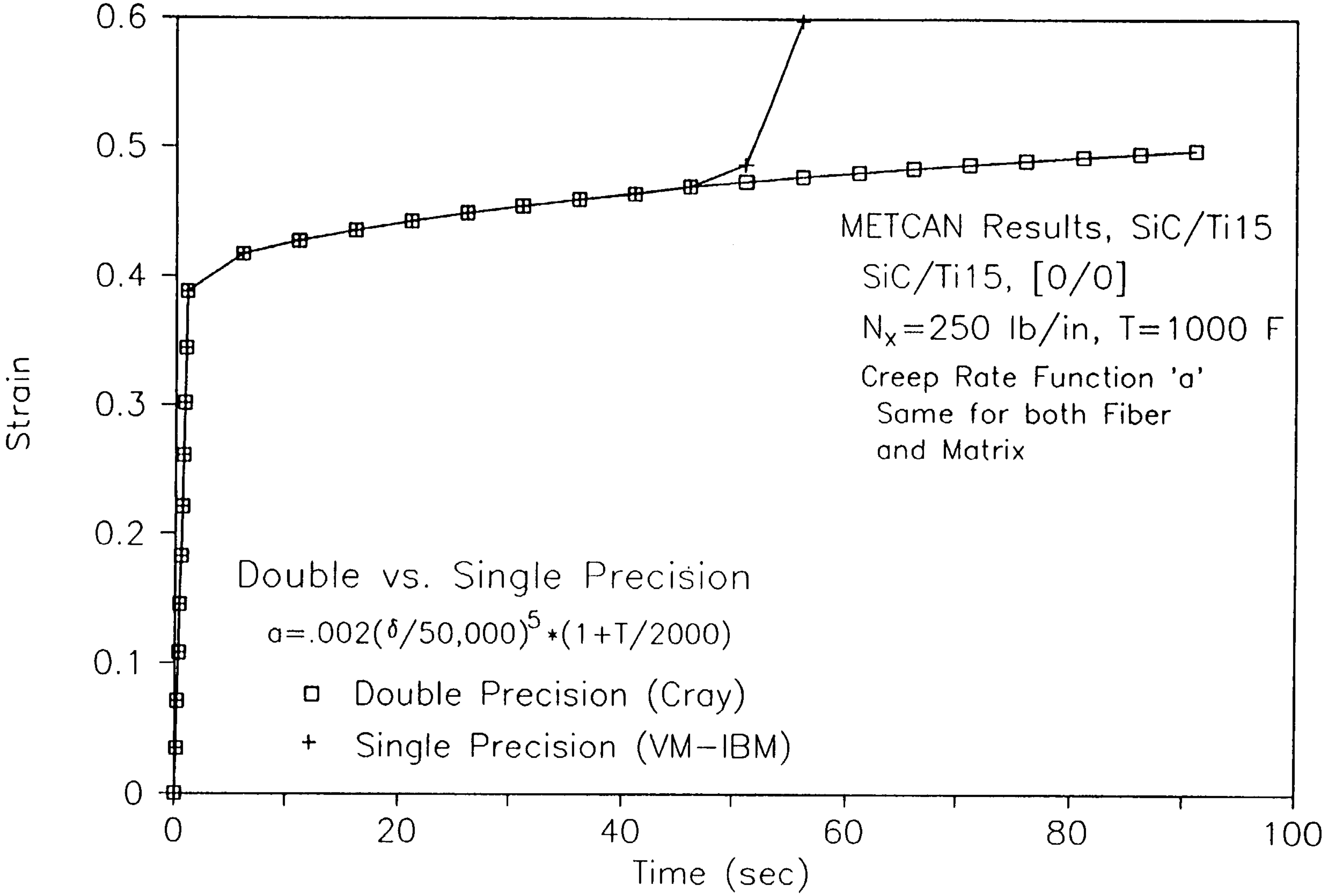


Fig. 16. Comparision of Double and Single Precision for Solving Nonlinear Creep Rate Materials.

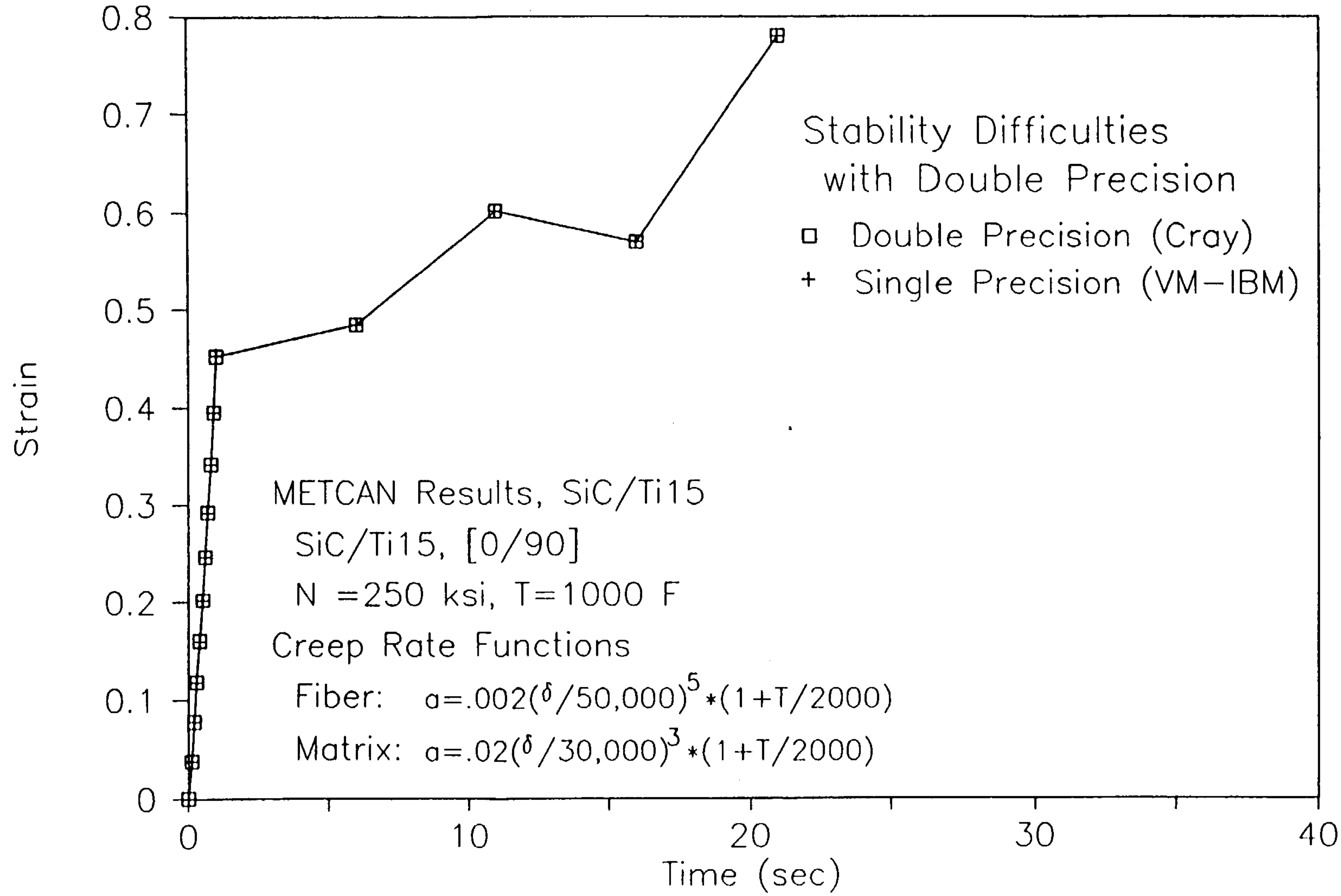


Fig. 17. METCAN Solution for a [0/90] Laminate.

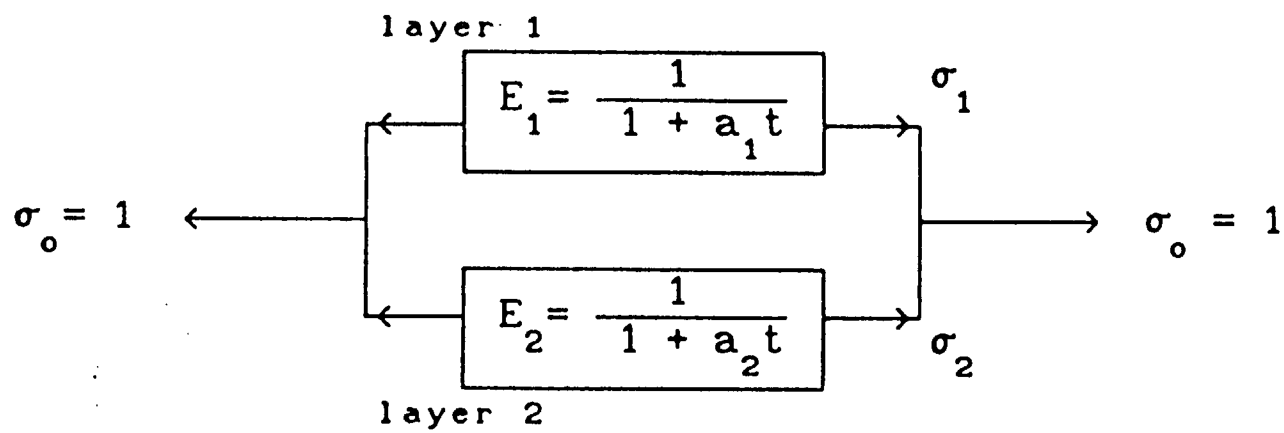


Fig. 18. One Dimensional Two Layer Material.

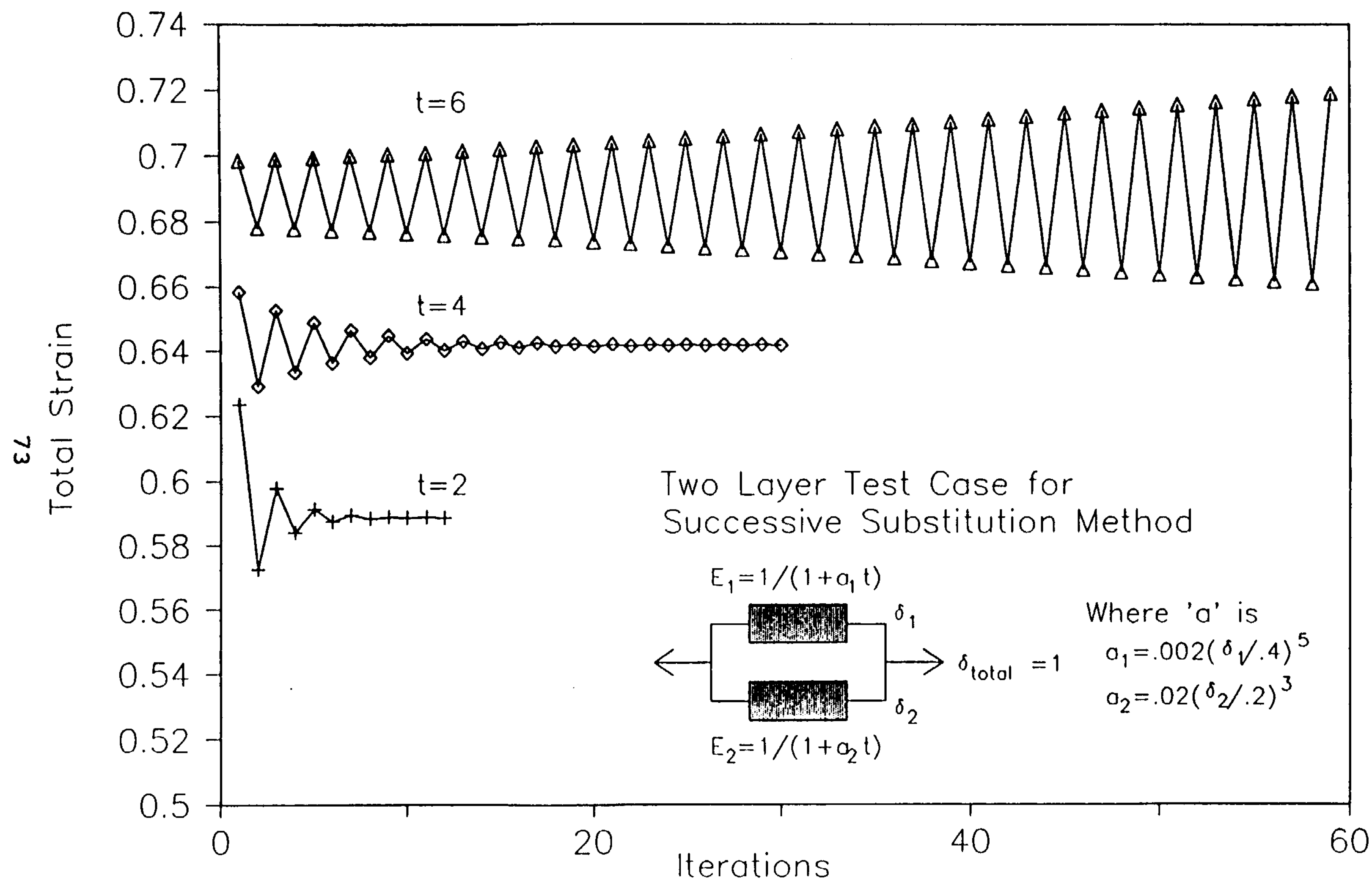


Fig. 19. Strain Results Using the Successive Substitution Method Method in METCAN for Time Step Size of 2.

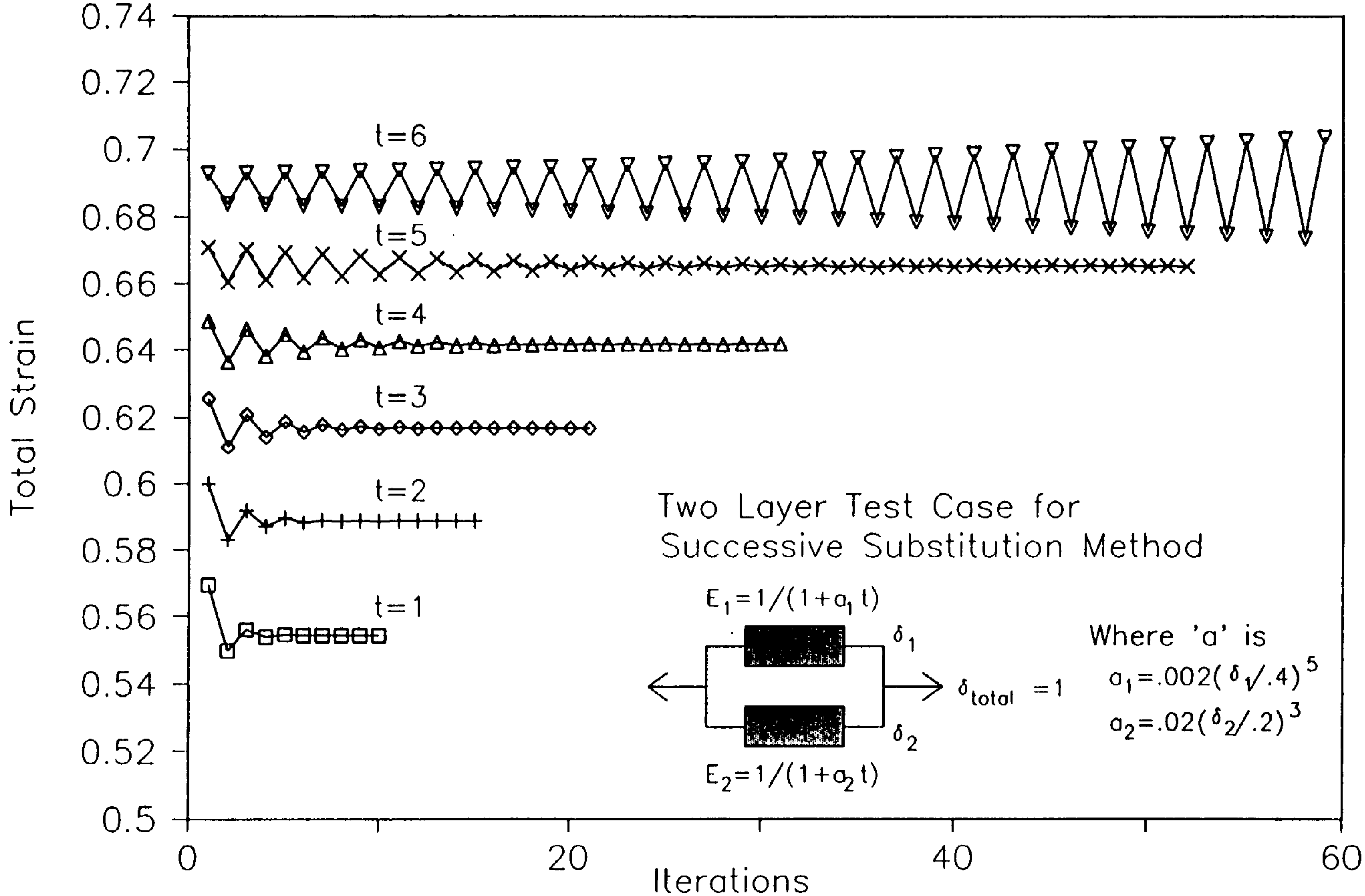


Fig. 20. Strain Results Using the Successive Substitution Method Method in METCAN for Time Step Size of 1.

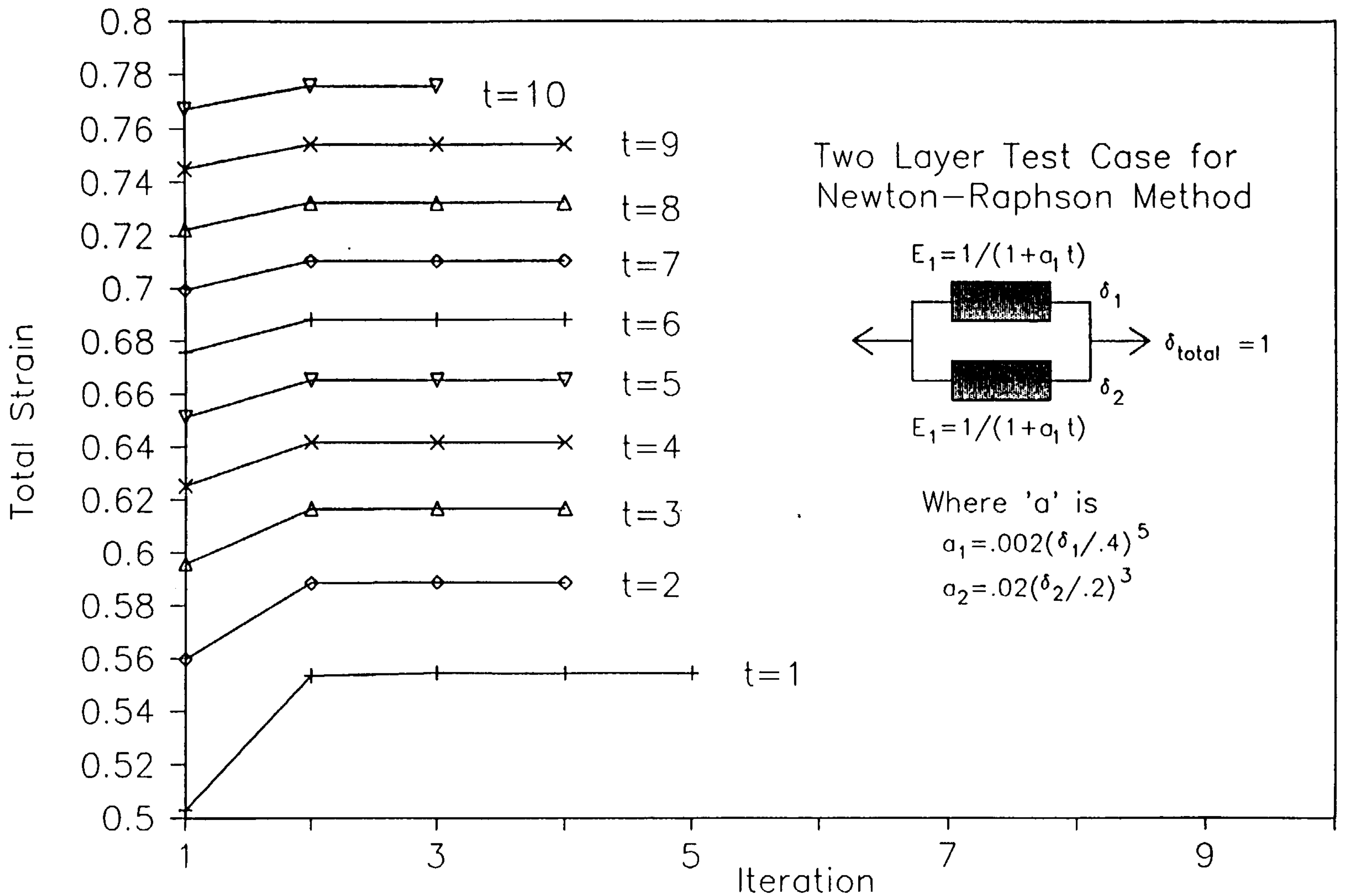


Fig. 21. Strain Results Using Newton–Raphson Method for Time Step Size of 1.

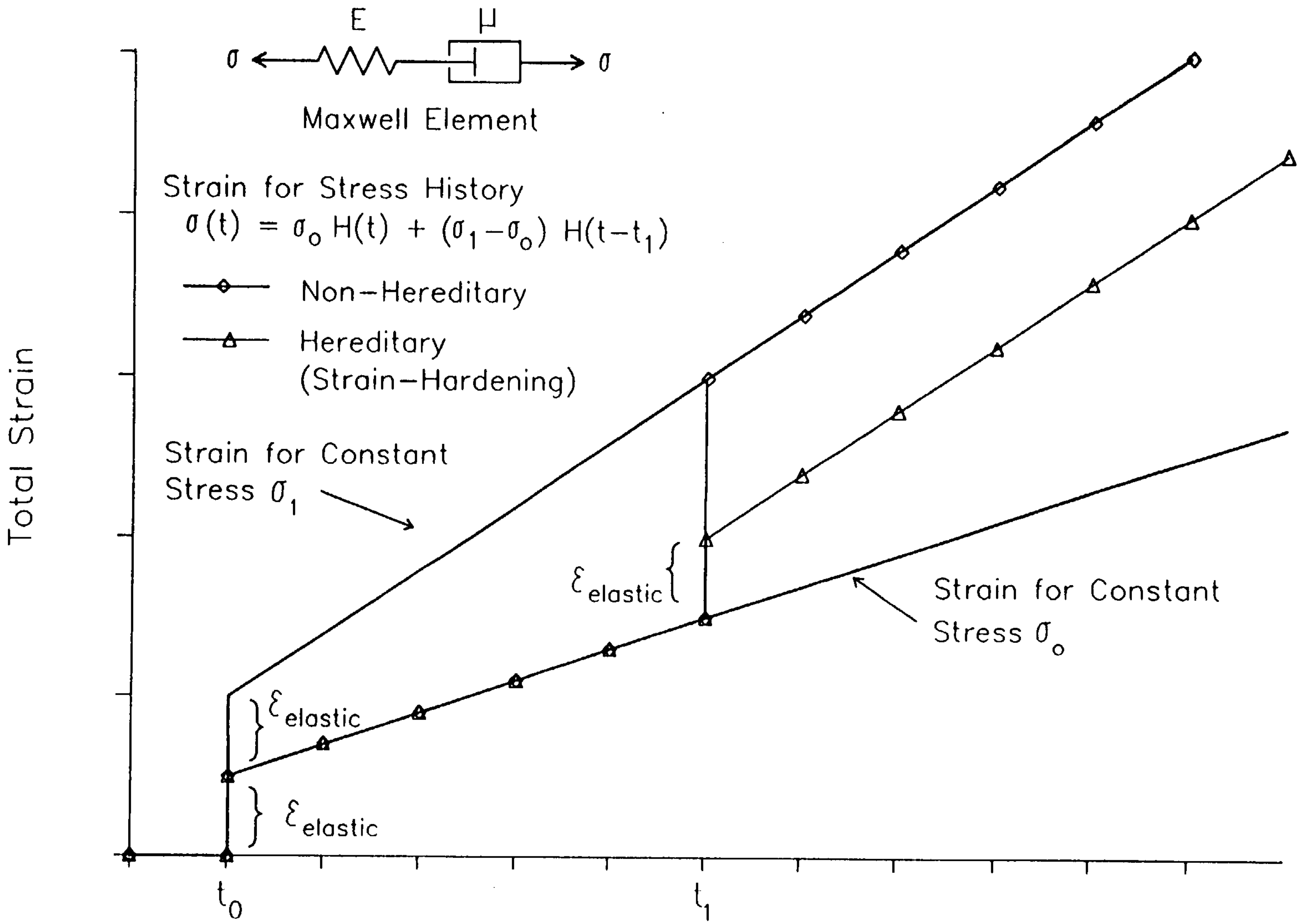


Fig. 22. Strain for Changing Stress Levels in Maxwell Element.

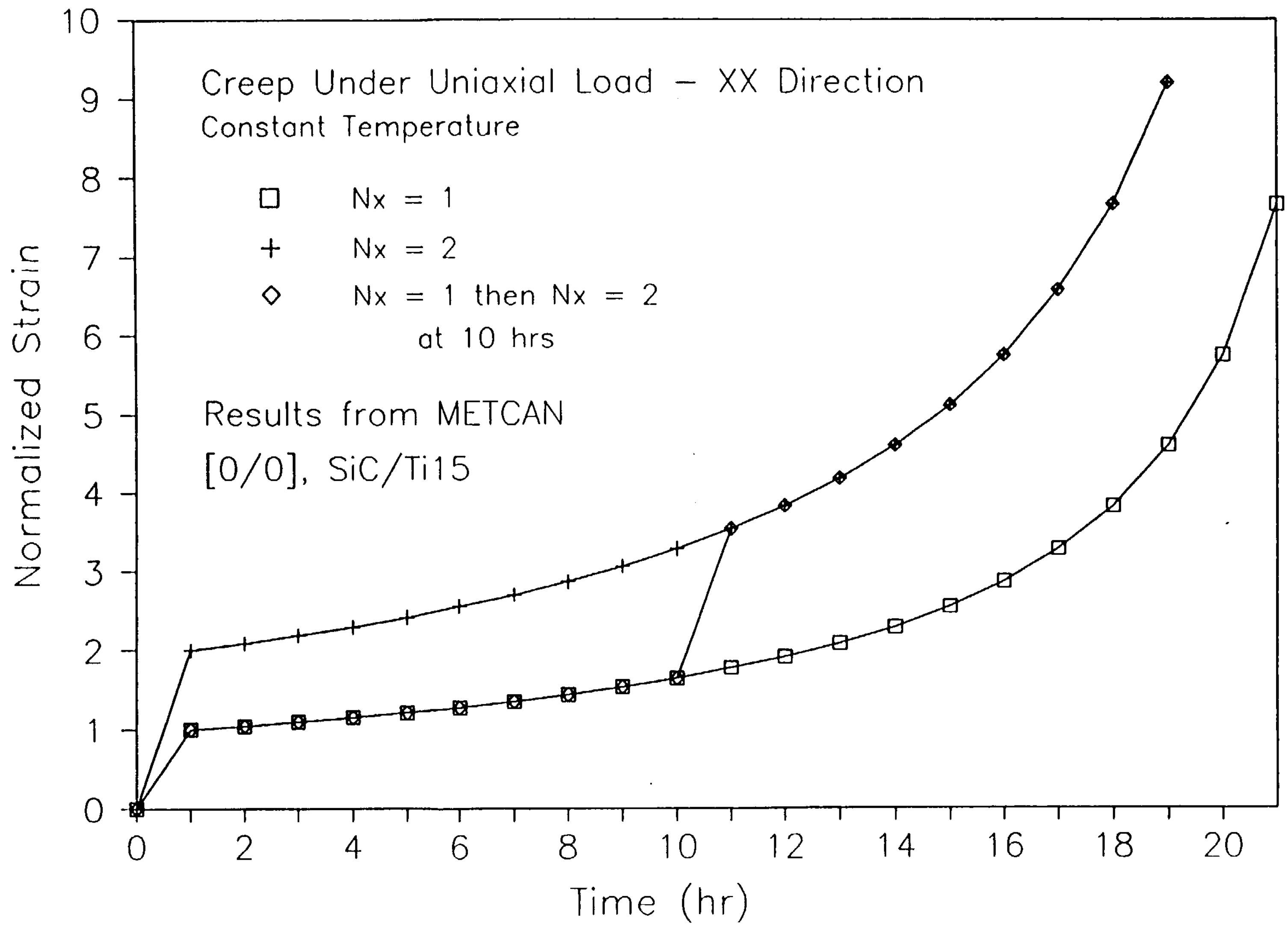


Fig. 23. Total Creep Strain for Constant Load.

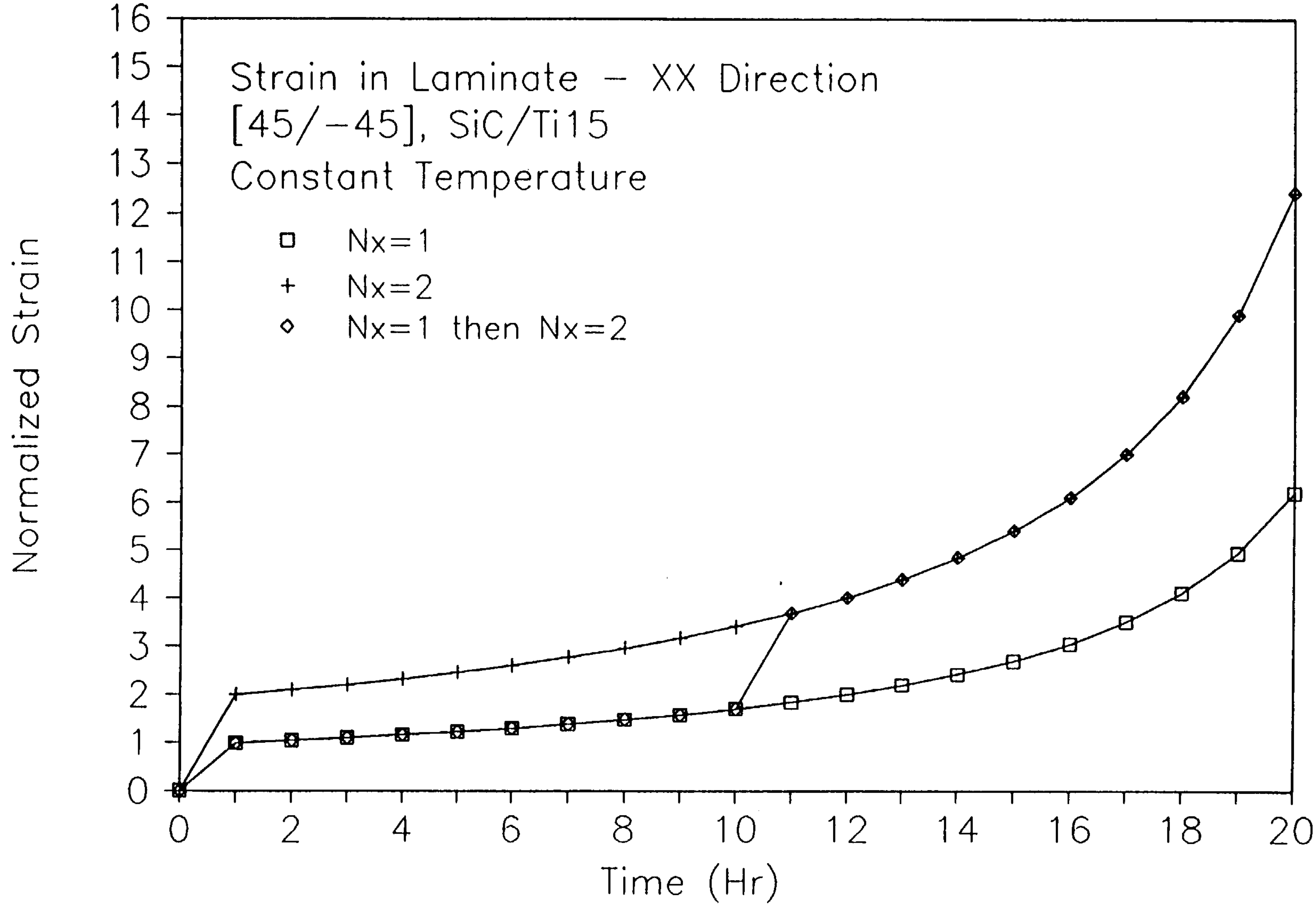


Fig. 24. Total Creep Strain for Constant Load.

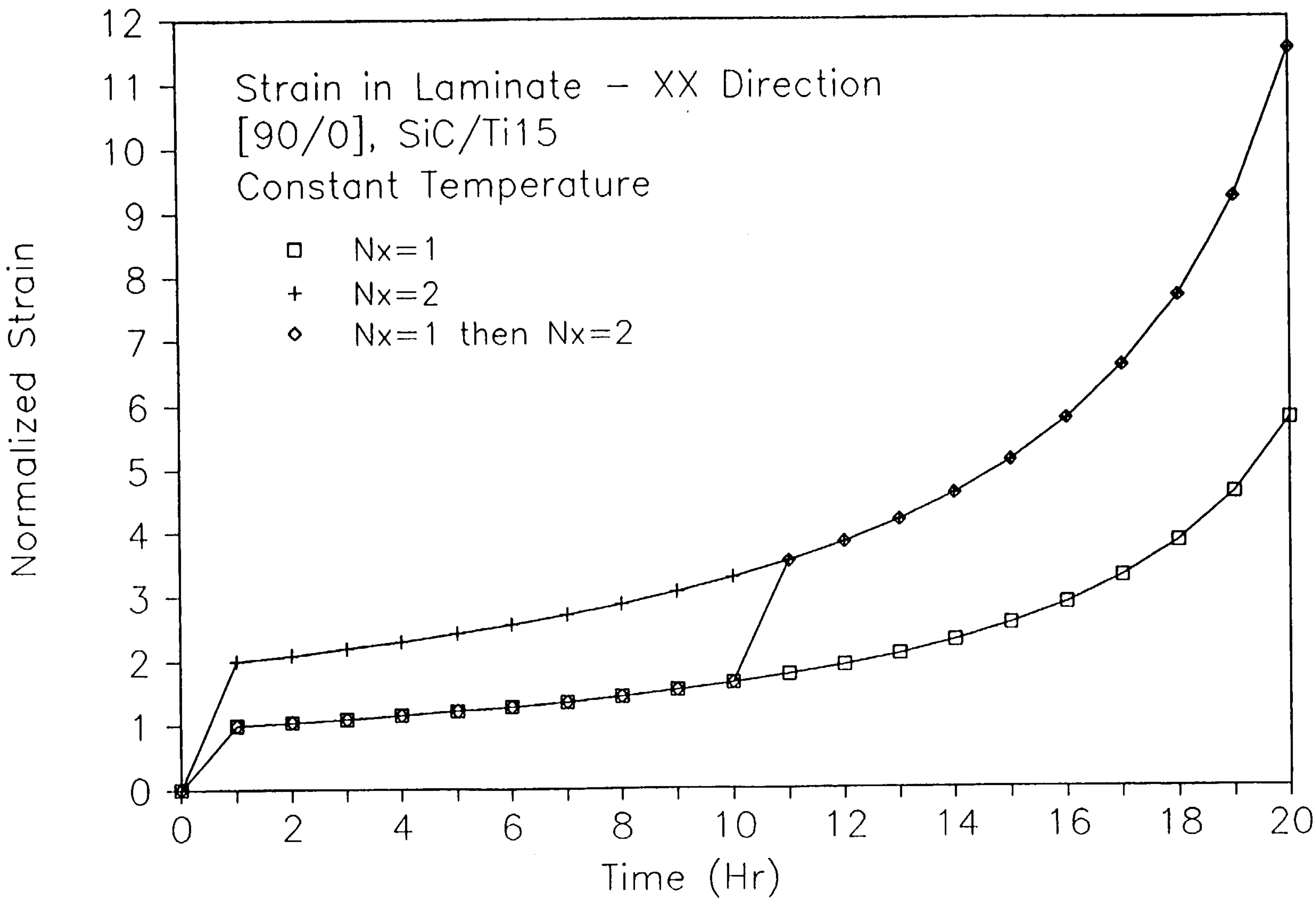


Fig. 25. Total Creep Strain for Constant Load.

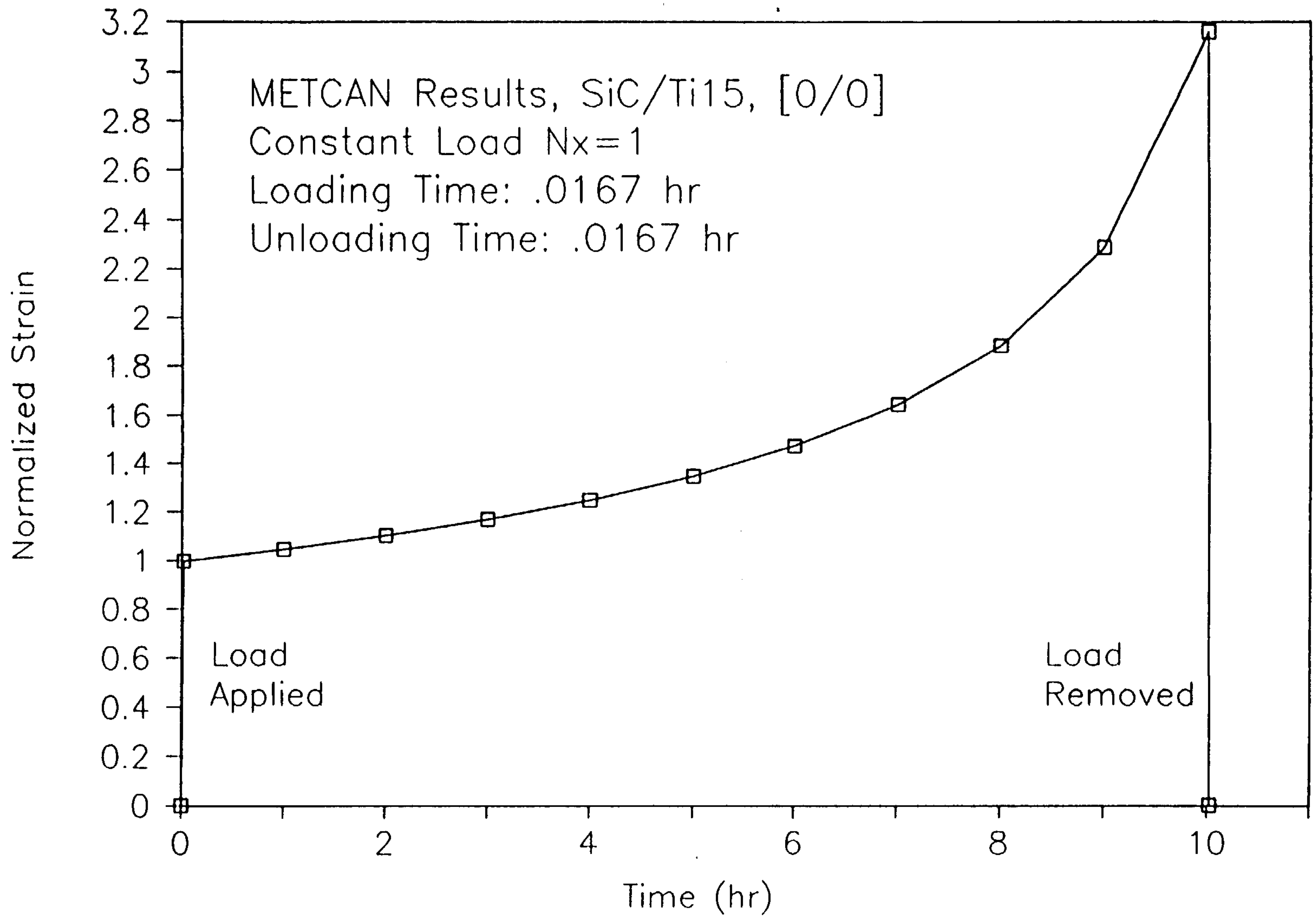


Fig. 26. METCAN Results for Constant Load with Unloading.

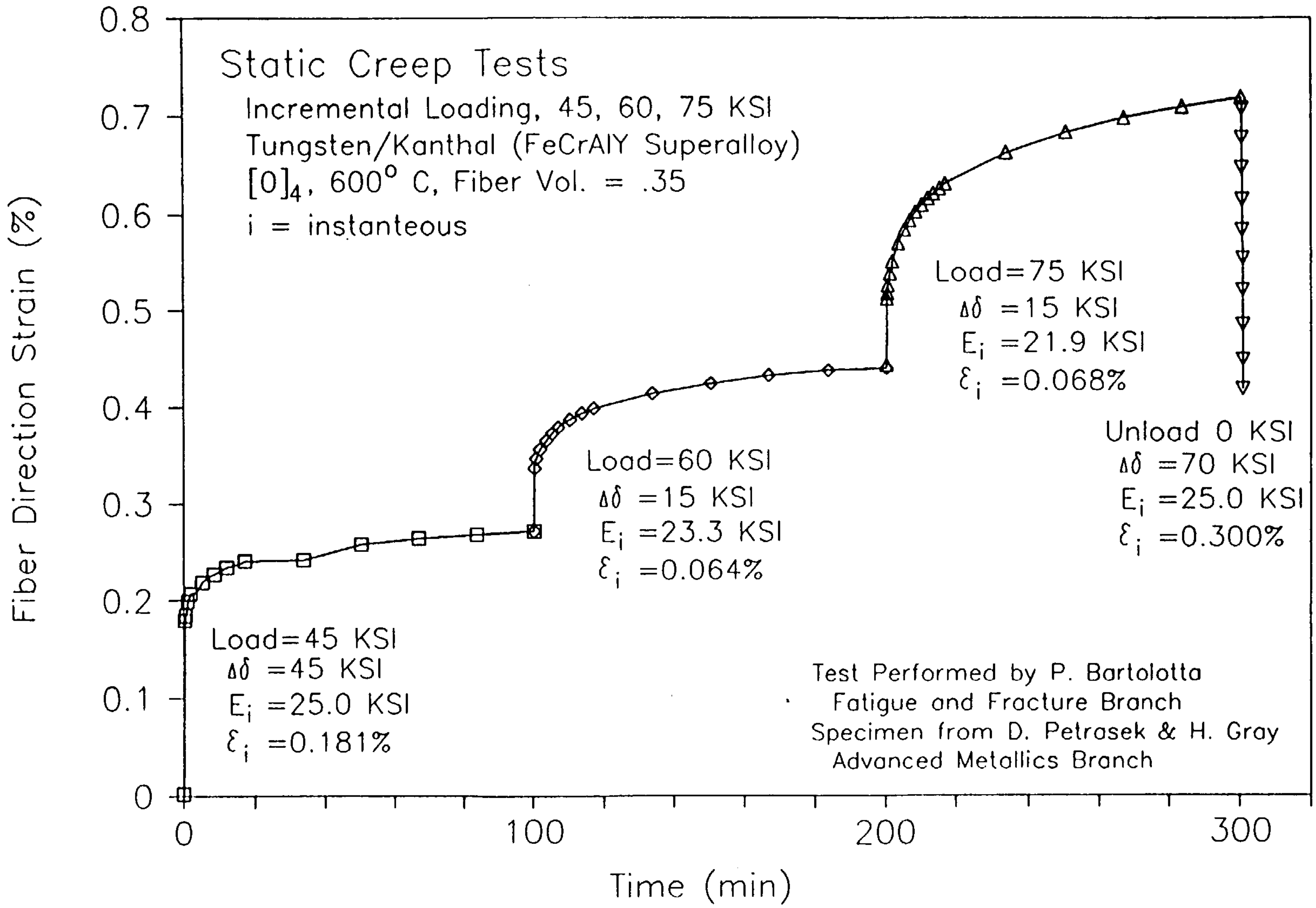


Fig. 27. Creep Tests on Tungsten/Kanthal at Three Load Levels.

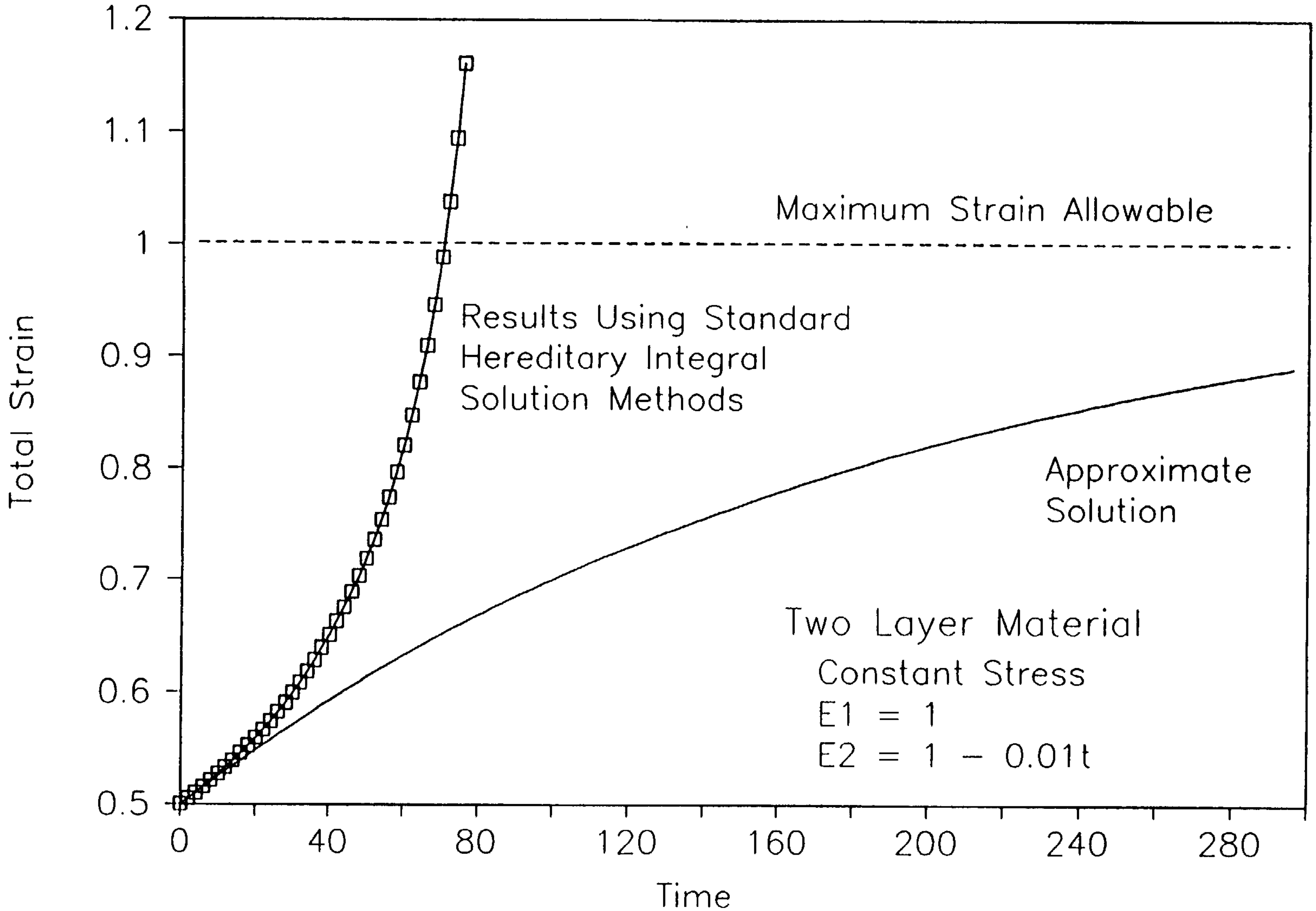


Fig. 28. Total Strain Due to Constant Stress on Two Layer Material

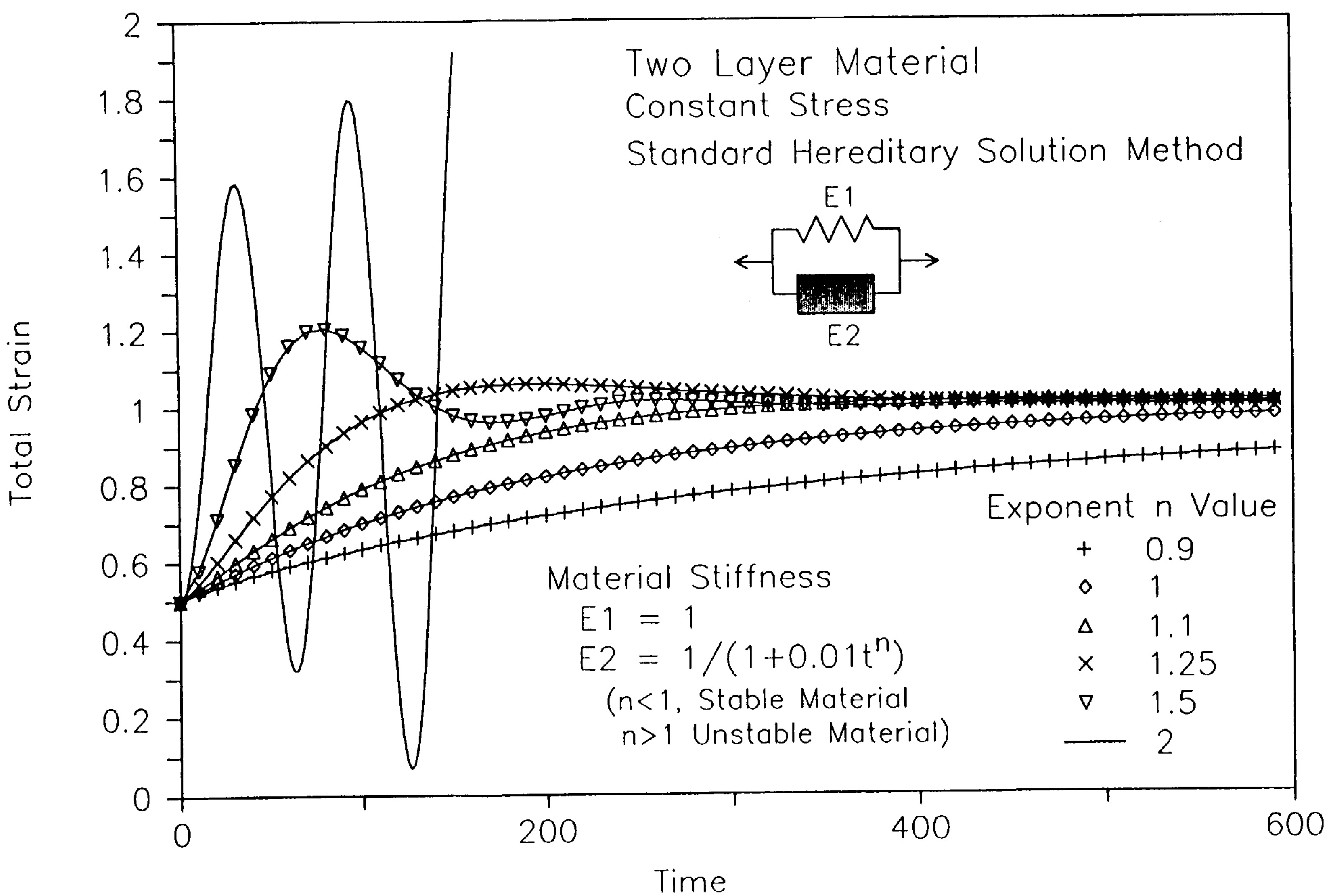
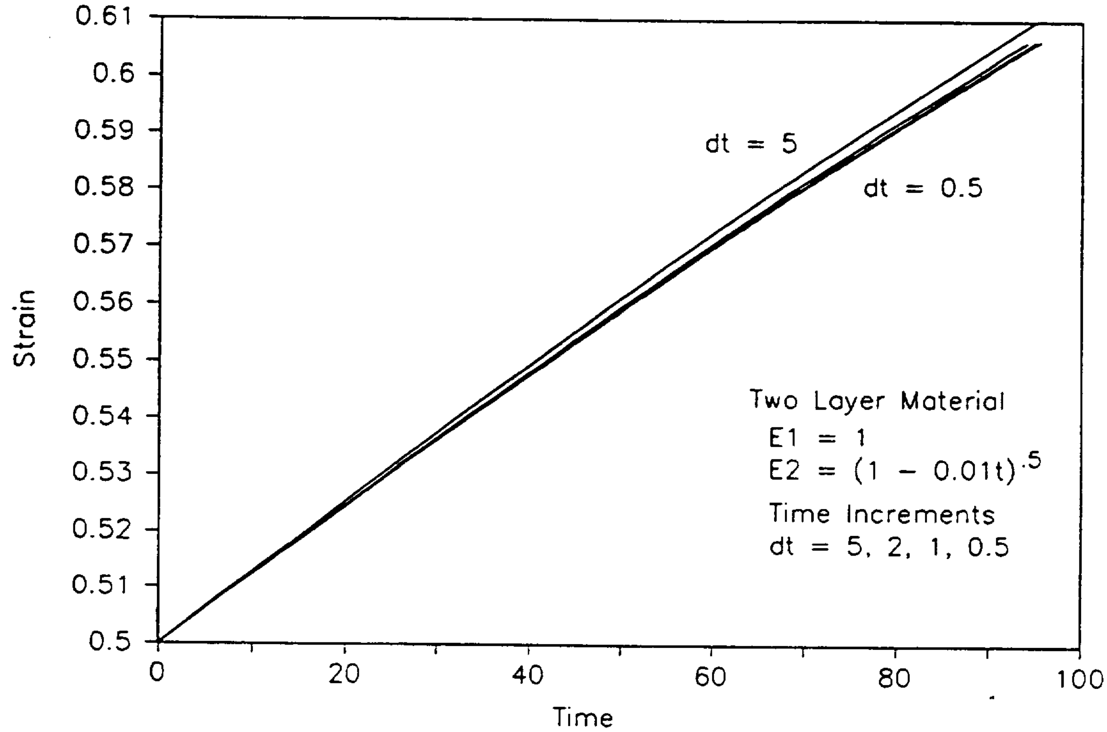


Fig. 29. Solution for Two Layer Material Using Standard Methods.



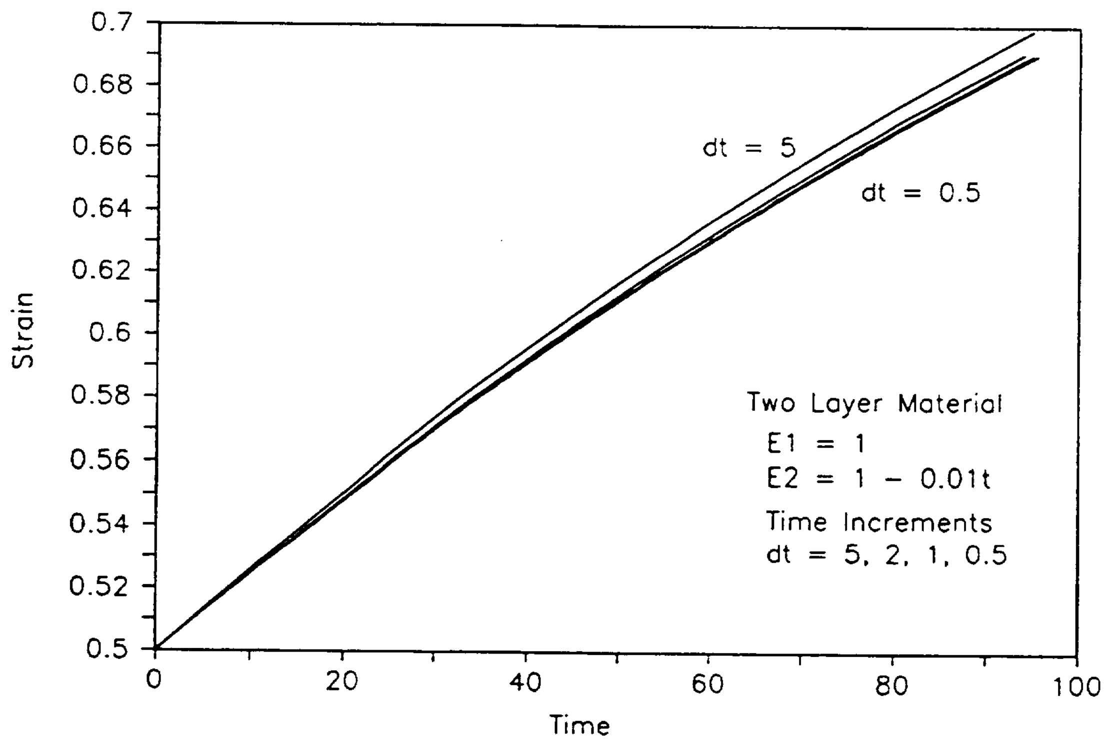


Fig. 30. Additive Solution Method for Two Layer Material at Different Time Increments.

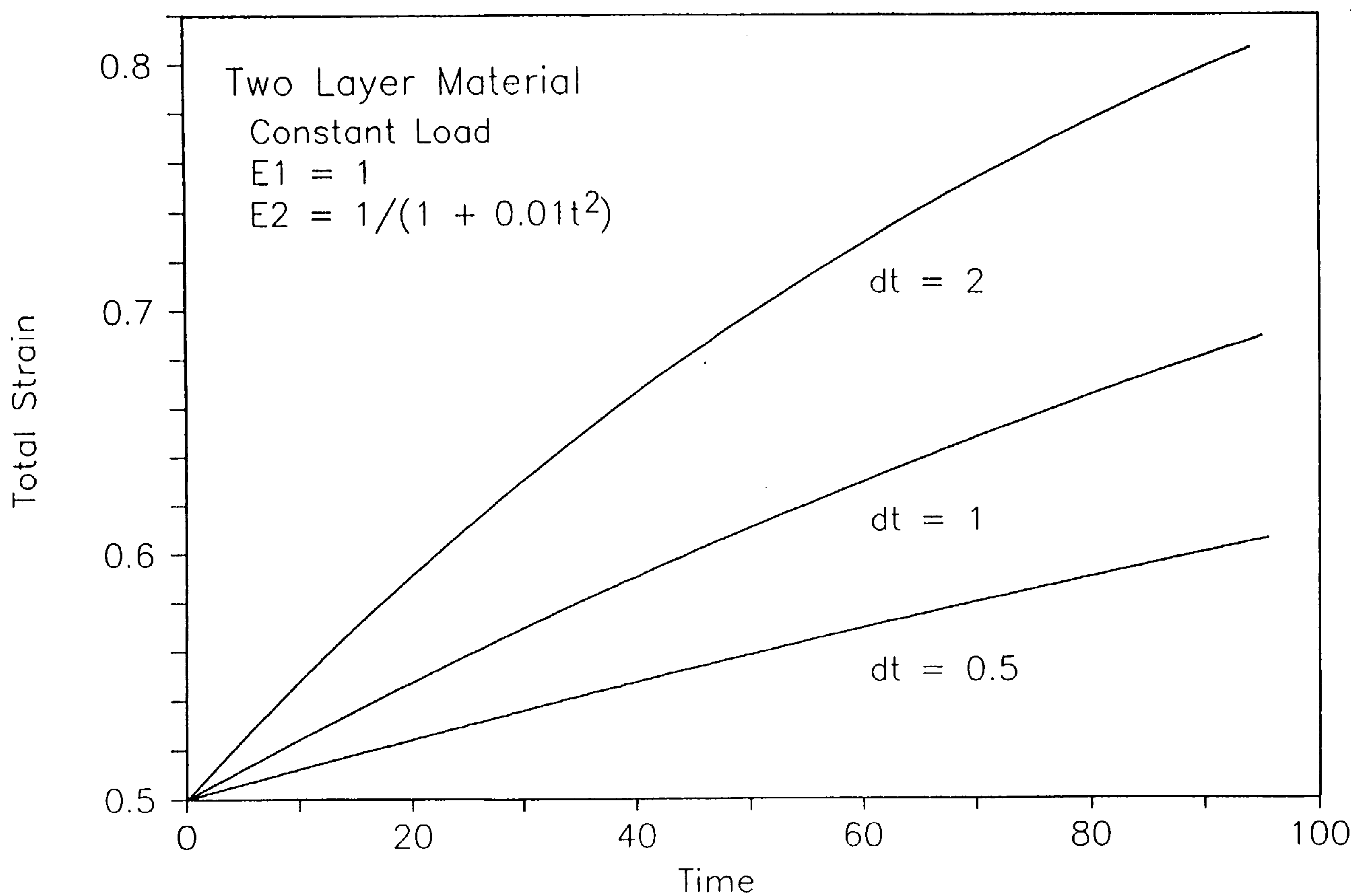


Fig. 31. Strain for Two Layer Material at Various Time Increments Using Additive Creep Method.

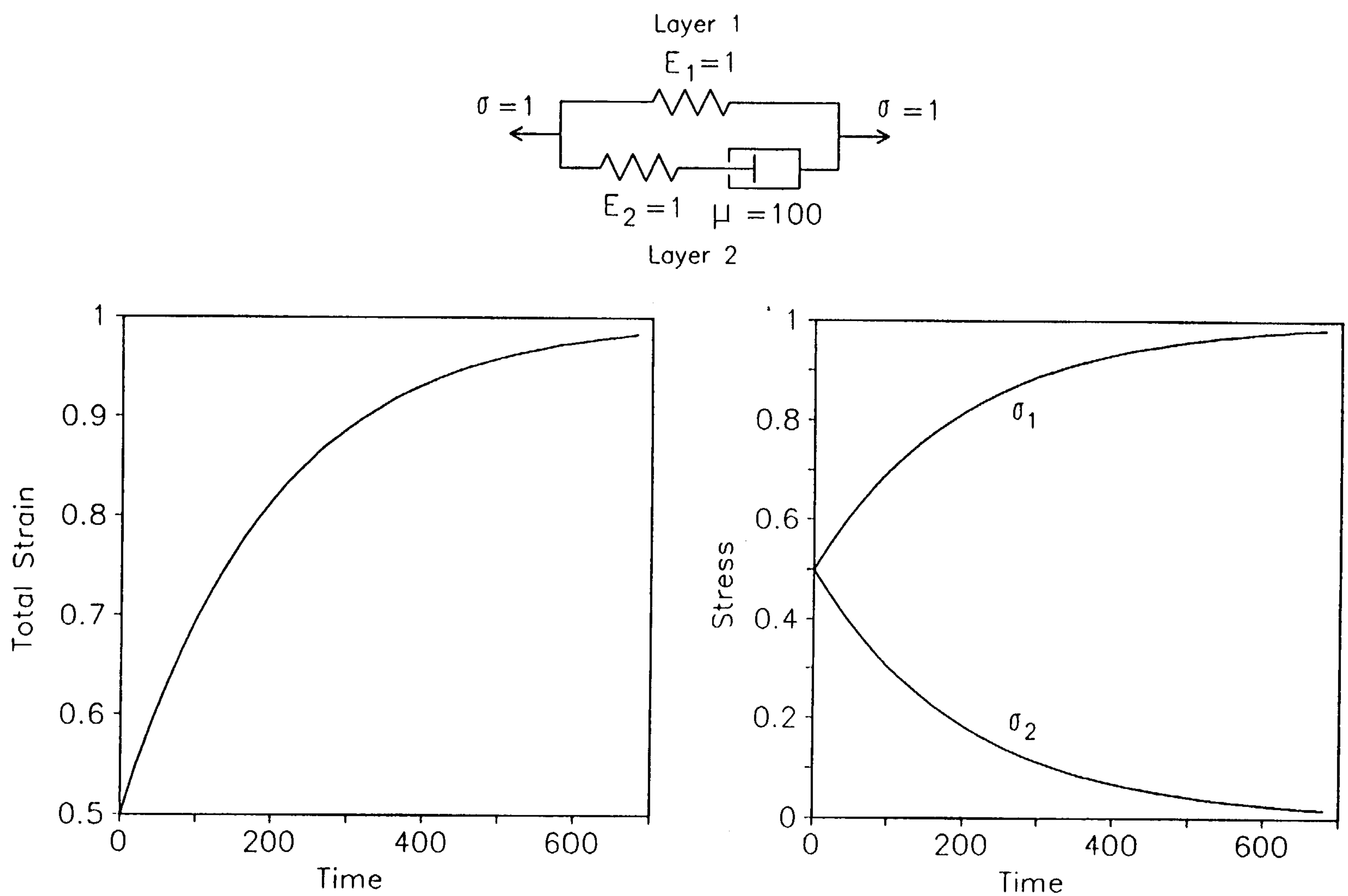


Fig. 32. Closed Form Solution of Linear Three Parameter Example.

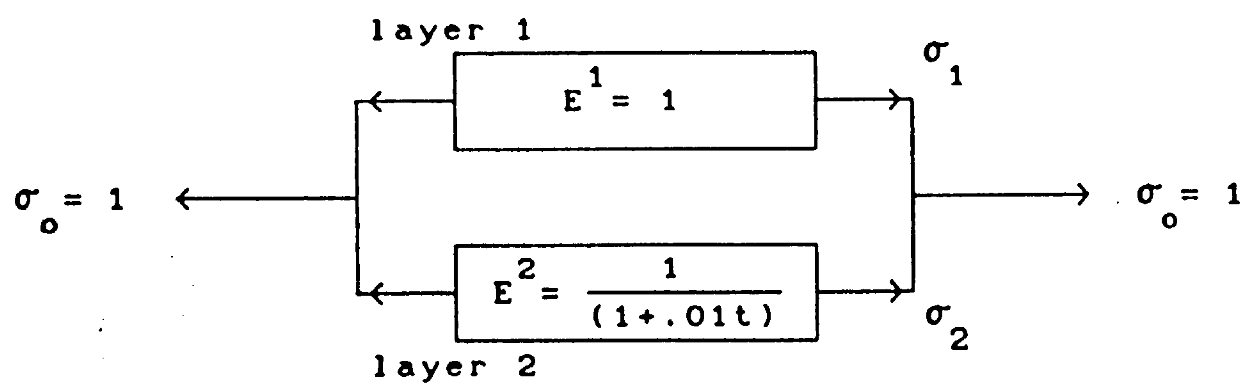


Fig. 33. Simplified Two Layer, One Dimensional Laminate.

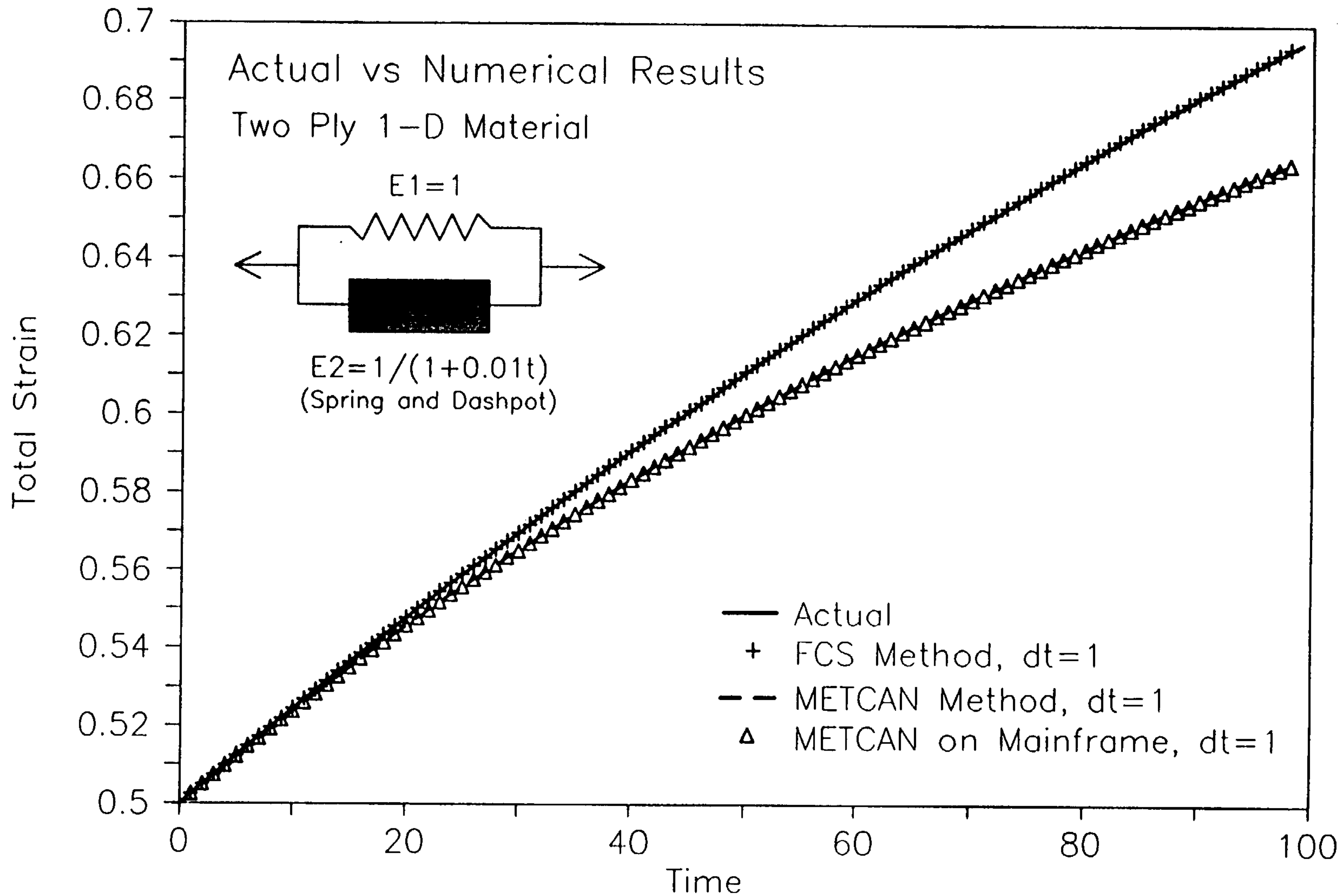


Fig. 34. Comparison of METCAN, Free Creep Strain Numerical Method and Actual Solution for 3 Parameter Model.

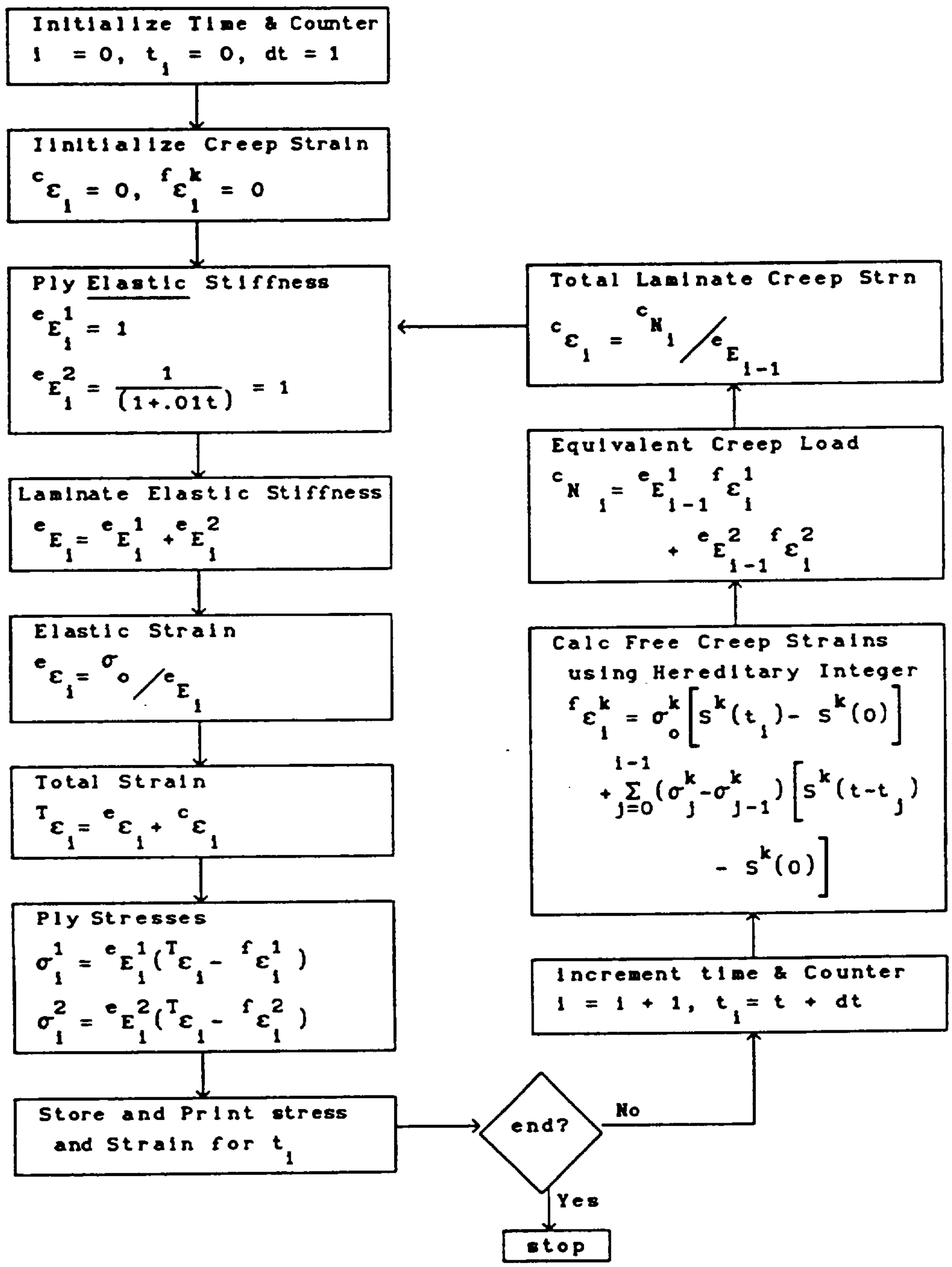


Fig. 35 Flowchart of FCS Method for Linear Three Parameters Example.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
August 1996

3. REPORT TYPE AND DATES COVERED
Final Contractor Report

4. TITLE AND SUBTITLE
Computer Simulation of Time-Dependent Effects for Metal Matrix Composites (MMC) Using METCAN

5. FUNDING NUMBERS
WU–505–63–5B
NGT36–027–011

6. AUTHOR(S)
Kurt C. Gramoll

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Memphis State University
Mechanical Engineering Department
Memphis, Tennessee 38152

8. PERFORMING ORGANIZATION REPORT NUMBER
E–10355

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135–3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR–198507

11. SUPPLEMENTARY NOTES
Project Manager, Christos C. Chamis, Structures Division, NASA Lewis Research Center, organization code 5200, (216) 433–3252.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category 24

This publication is available from the NASA Center for AeroSpace Information, (301) 621–0390.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***

The METCAN (METal matrix Composite ANalyzer) computer code and its underlining theory, including the Multi-Factor Interaction (MFI) equation, were examined for time-dependent response of metal matrix composites (MMC). This study concentrated on modeling time effects for fiber and matrix material properties, particularly for the modulus, and the respective creep response due to thermomechanical loading. The four main concepts addressed were, one, modeling of the three basic stages of creep, two, implementation of the modified MFI equation, three, characterization of in-situ material properties, and four, numerical methods for simulating viscoelastic creep. The difficulty of experimentally obtaining the numerous in-situ material properties for use in METCAN is discussed and two possible alternatives are presented.

14. SUBJECT TERMS
Micromechanics; Constituent materials; Interaction effects; Material models; In-situ properties; Numerical methods; Computer code; Thermomechanical loading and creep response

15. NUMBER OF PAGES
91

16. PRICE CODE
A05

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	